PROJECT IN SUMMER SCHOOL 2009 IN 'PPISR'

Name of the Project:

GENERAL RELATIVISTIC SPIN PRECESSION OF BINARY PULSARS

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Aim:

To calculate the rate of the 'Precession of The Equinoxes' using 'Newtonian Gravity' & that of a Pulsar in a Binary System using 'The Theory of General Relativity'.

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OVERVIEW OF MY CONTENTS

A. INTRODUCTION :

As we know if a body is perfectly spherical in shape, then according to Newtonian Gravity, there is no net external torque acting on it due to other external objects. So the angular momentum of the perfectly spherical body is conserved.

But if we handle this problem under the 'General Theory of Relativity', then we will see that though the body is a perfectly spherical one, even then there is a net external torque exerted on it due to other external objects!

To discuss these I will consider two astronomical examples: one is 'Precession of the Equinoxes' and the other one 'Pulsars in a binary system'.

1. As we know the distance between the earth and sun is ~ 15×10^8 km, which is comparatively very large, so to discuss the effect due to the sun, we can treat this problem under Newtonian Gravity and also for the effect because of the moon, again this can be handled under Newtonian Gravity as the moon is not a very massive one.

Due to spinning [3] of the earth on its own axis, the shape of the earth is not perfectly spherical but a closely approximated oblate spheroid. This means that there is a bulging portion near the equatorial region of the earth and it can be shown that the difference between the equatorial radius and polar radius is approximately 11.12 km.

Due to this deviation in shape from perfectly spherical and also as the earth's axis of rotation is inclined at an angle ~$66°33'$ with respect to the plane of the ecliptic (orbital plane of the earth), therefore the gravitational force of the sun gives rise to a net external torque. This torque causes the spin axis to precess about the normal of the ecliptic. This astronomical phenomenon is known as the 'Precessions of the Equinoxes'.

During one half of the earth's orbital period, the part of the bulge above the ecliptic (A) is nearer to the sun than the lower part (B). The mass at A is therefore attracted more strongly by the sun than that of mass at B. So the equatorial bulge of the earth experiences a couple that acts perpendicular to the

[3] Actually 'Spin' is purely a relativistic quantum mechanical concept. It is an intrinsic property of a particle. This is totally different from our classical orbital angular momentum concept. There is no classical analog of 'Spin'. Spinning of earth respect to its own axis is actually orbital angular momentum of constituent point particles respect to its axis.

plane formed by the earth's axis of rotation and the normal to the plane of the ecliptic[counter clockwise torque on the earth, out of the plane of the figure] (fig1).

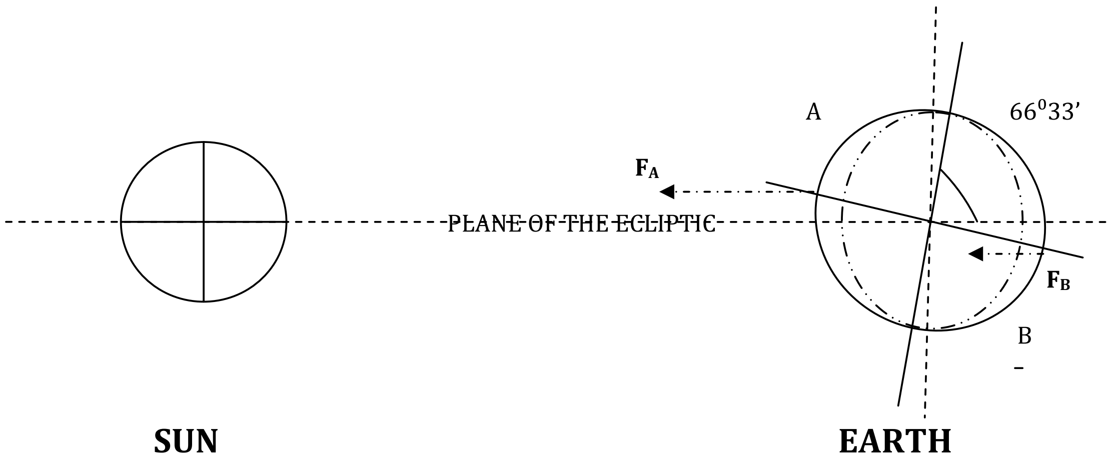


fig 1

Six months later when the earth is on the other side of the sun, the mass at B is attracted more strongly than that of mass at A (fig 2). However the torque has the same direction in space as before. Midway between these extremes, the

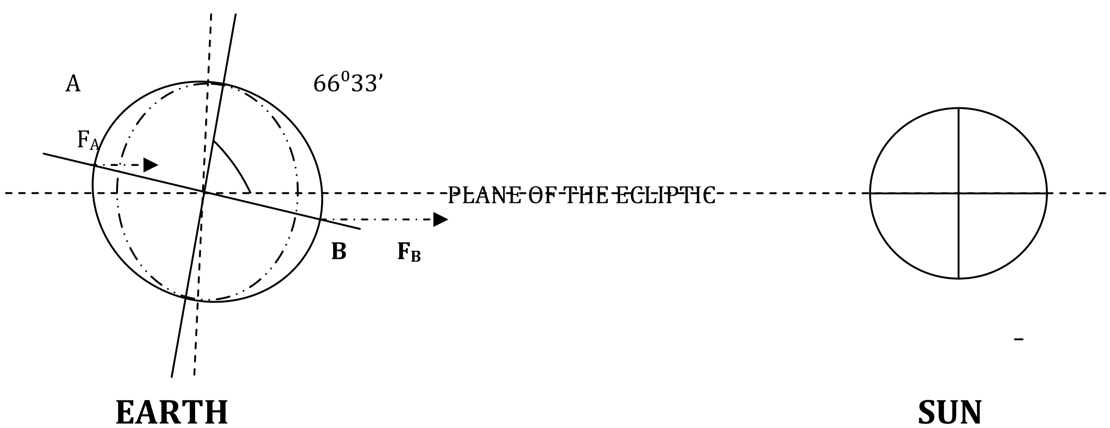
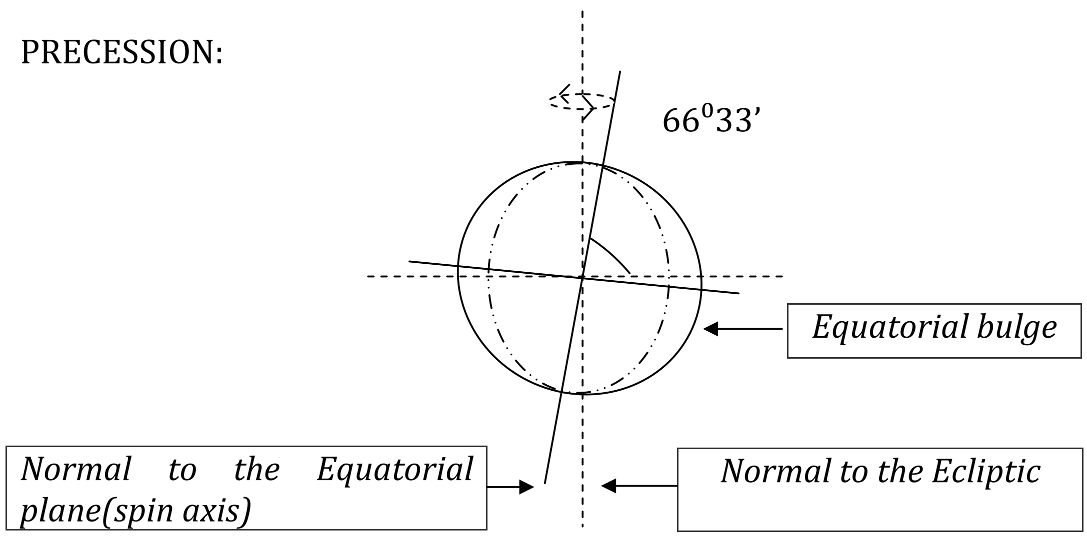


fig 2

torque is zero. The average torque is perpendicular to the spin angular momentum and lies in the ecliptic plane. In a similar way the moon also exerts an average torque on the earth; this torque is about twice as great as that due to the sun. Due to these torques the spin axis of the earth does precess slowly but steadily (about the normal to the ecliptic), keeping of course, the angle between the spin-axis of the earth and the plane of the ecliptic constant. The period of this precession is approximately 26,000 years.

PRECESSION:



EARTH

fig 3

2. For the second part I am considering pulsars in a binary system. Since the pulsars are believed to be rapidly rotating Neutron stars, the system can be thought of as the gravitational analog of a semi-relativistic atom with spin. As any binary system is characterized by two objects of comparable masses and in case of binary pulsar system, as individual pulsar has huge mass (>1.4 Solar mass) and the distance between these two is comparatively less, so the usual test particle assumption is not valid and so we must have to handle this problem with proper recoil corrections taken into account i.e. the problem should be treated under the ‘Theory of General Relativity’ instead of Newtonian Gravity.

To treat this problem under General Relativity we can use Gupta’s quantum theory of gravity. In this process we consider the precession equation for the spin vector of a ‘Dirac Particle’ in the gravitational field and then we generalize this to the macroscopic spinning objects. But there are some serious drawbacks, which are involved in this approach. Because we know that the spin (intrinsic) has no classical analog, and hence any classical limit of a quantum equation that retains the spin term has to be carried out with great care. Now to interpret for the macroscopic object, if we think the macroscopic object as due to a large collection of spin ½ particles with a total macroscopic spin, then also it is a bit deceptive as for such a system, the angular momentum per unit mass goes to zero in classical limit.

But if we follow the purely classical, but non-geometrical derivation of the interaction energy for the semi-relativistic gravitational two-body system having arbitrary spins which is given by Prof. C.F.Cho and Prof. N. D. Hari Dass using Schwinger's elegant Source Theory [5] approach, then we will see, though we consider the two Pulsars in a binary system as perfectly spherical one, then also the spin axis of individual pulsar does precess around the normal of the orbital plane of the two pulsars passing through the center of mass of them. Whereas according to Newtonian Gravity this should not precess at all. The double pulsars 'PSR J0737-3039 A/B' consists of two Neutron stars in a highly relativistic orbit that displays a relativistic spin precession rate of $4.77° \pm 0.66°$ per year.

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[5] 'Source Theory' is basically a theoretical description of particle interaction unlike geometrical viewpoint of the 'Theory of General Relativity' or operator approach of the 'Quantum Field Theory'.

B. **<u>PRECESSION OF THE EQUINOXES</u>**

As the shape of the earth is mainly responsible for this astronomical phenomenon, so at first I want to figure out the approximated shape of the earth.

- *SHAPE OF THE EARTH*

The surface of the earth is an equipotential surface because if it were not an equipotential one, then there would be a component of the gravitational force along the surface, due to which mass should flow from the higher potential region to the lower one. But in our practical experience we never see this phenomenon. Therefore, we can conclude the surface of the earth must be an equipotential one [(6)].

Mainly I am interested in the difference between the equatorial radius and polar radius. Since only by this the approximate shape of the earth can be figured out.

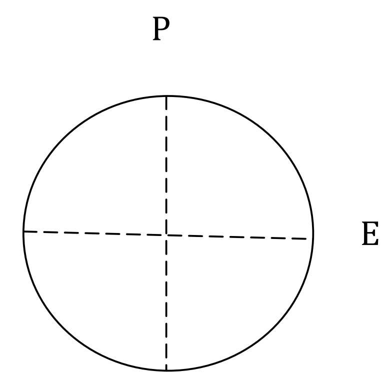
potential at P=potential at E

$V(P)=V(E)$

$\Rightarrow V_g(P)+V_c(P)=V_g(E)+V_c(E)$

where $V_g=$ *gravitational potential*

$V_c=$ *potential corresponding to* centrifugal force



EARTH

[E≡ *equatorial* & P≡ *polar*]

fig 4

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(6) Gravitational force, $\vec{F}=-\vec{\nabla}V$ where, V≡ *gravitational potential*

If the surface is an equipotential one, then $dV=\vec{\nabla}V.\vec{dr}=0$ i.e. $\vec{\nabla}V$ is perpendicular to $\vec{dr}$. As $\vec{dr}$ is along the tangent plane at any point on the surface of the earth, so $\vec{\nabla}V$ i.e. $\vec{F}$ is perpendicular to the tangent plane drawn through the point on the surface.

So,

$$-\frac{GM_E}{r_P}-\frac{\omega^2 r_{\perp_P}{}^2}{2}=-\frac{GM_E}{r_E}-\frac{\omega^2 r_{\perp E}{}^2}{2}$$

where,

$G =$ *universal gravitational constant*

$=6.67 \times 10^{-11}\ Nm^2kg^{-2}$ (S.I.)

$M_E =$ *mass of the earth*

$= 5.98 \times 10^{24}\ kg$

$r_P =$ *polar radius*

$=6.4 \times 10^6\ m$

$r_E =$ *equatorial radius*

$\omega =$ *angular velocity of the earth respec to its own axis*

$=\frac{2\pi}{24\times 60\times 60} rad/s$

$=7.727 \times 10^{-5}\ rad/s$

$r_{\perp_P} =$ *perpendicular distance from axis of rotation to the pole*

$=0$

$r_{\perp_E} =$ *perpendicular distance from axis of rotation to the equator*

$= r_E$

NOTE: centrifugal acceleration, $a_c = \omega^2 r_\perp = -\frac{dV_C}{dr_\perp} \Rightarrow \int_0^{V_C} dV_C = -\omega^2 \int_0^{r_\perp} r_\perp dr_\perp \Rightarrow \boxed{V_C = -\frac{\omega^2 r_\perp{}^2}{2}}$

$r_\perp$ =perpendicular distance from the axis of rotation. So in North & South Pole V_c=0 and in Equator

Now, let $r_P = r_E(1-\in)$

$$\Rightarrow r_E = \frac{r_P}{1-\in}$$

So, $\frac{GM_E}{r_P} = \frac{GM_E}{r_E} + \frac{\omega^2 r_E^2}{2}$

$$\Rightarrow \frac{GM_E}{r_P} = \frac{GM_E}{r_P}(1-\in) + \frac{\omega^2 r_P^2}{2(1-\in)^2}$$

$$\Rightarrow \frac{GM_E}{r_P} \approx \frac{GM_E}{r_P} - \frac{GM_E}{r_P}\in + \frac{\omega^2 r_P^2}{2}(1+2\in)$$

$$\Rightarrow \frac{GM_E}{r_P}\in \approx \frac{\omega^2 r_P^2}{2}$$ [As $\in << 1$, $\frac{1}{(1-\in)^2} = (1-\in)^{-2} \approx (1+2\in) \approx 1$]

$$\Rightarrow \boxed{\in \approx \frac{\omega^2 r_P^3}{2GM_E}}$$

$$\Rightarrow \in \approx \frac{(7.727\times10^{-5})^2(6.4\times10^6)^3}{2\times6.67\times10^{-11}\times5.98\times10^{24}}$$

$$\Rightarrow \boxed{\in \approx 1.738\times10^{-3}}$$

Now,

$$r_P = r_E(1-\in)$$

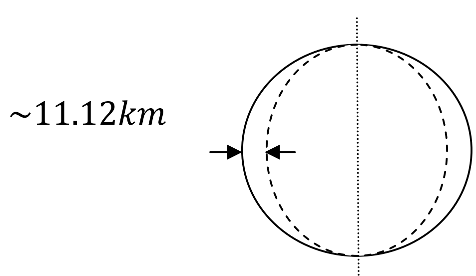
$$\Rightarrow r_E \approx r_P(1+\in)$$

$$\Rightarrow r_E - r_P \approx \in r_P$$

$$\Rightarrow r_E - r_P = h \approx (1.738\times10^{-3})(6.4\times10^6)m$$ [as $\in << 1$]

$$\Rightarrow \boxed{h \approx 11.12\times10^3 m = 11.12 km}$$

So, the difference between equatorial and polar radius is $\sim 11.12\ km$



EARTH $\boxed{fig\,5}$

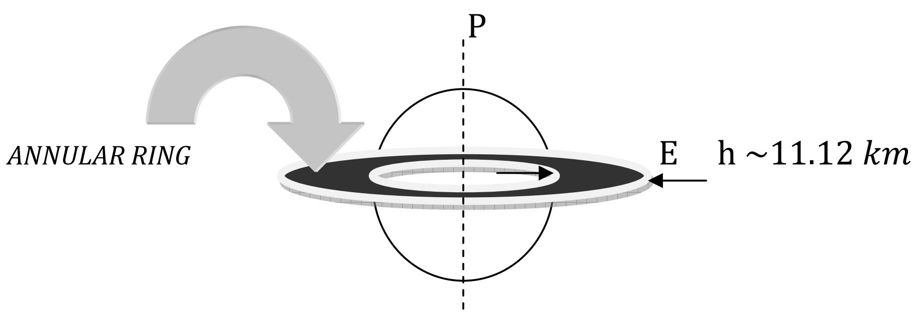
So, due to spinning of the earth about its own axis, the shape of the earth becomes an approximated oblate spheroid instead of a perfectly sphere. The difference between the equatorial and polar radius is $\sim 11.12\ km$.

- *MOMENT OF INERTIA OF THE EARTH*

As we will see to calculate the precession rate we require the moment of inertia of the earth with respect to its three principal axes, so now I want to calculate the moment of inertia of the earth.

Exact calculation of the moment of inertia of the earth is much more complicated due to its shape. Here what I will do is an approximate one.

I will approximate the earth as a solid sphere having uniform mass density with an annular ring around the equatorial region of the earth.



APPROXIMATED EARTH

fig 6

Now, let me define a right-handed Cartesian co-ordinate system for the earth.

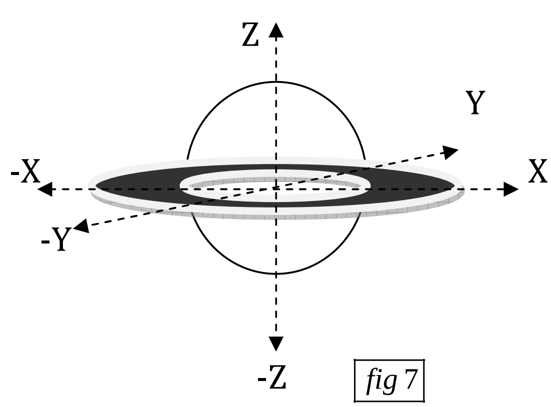


fig 7

So, X & Y axes are defined on the equatorial plane and Z is perpendicular to the equatorial plane & passing through two poles.

Let,

Moment of inertia of the earth about X-axis= I_X

Moment of inertia of the earth about Y-axis= I_Y

Moment of inertia of the earth about Z-axis= I_Z

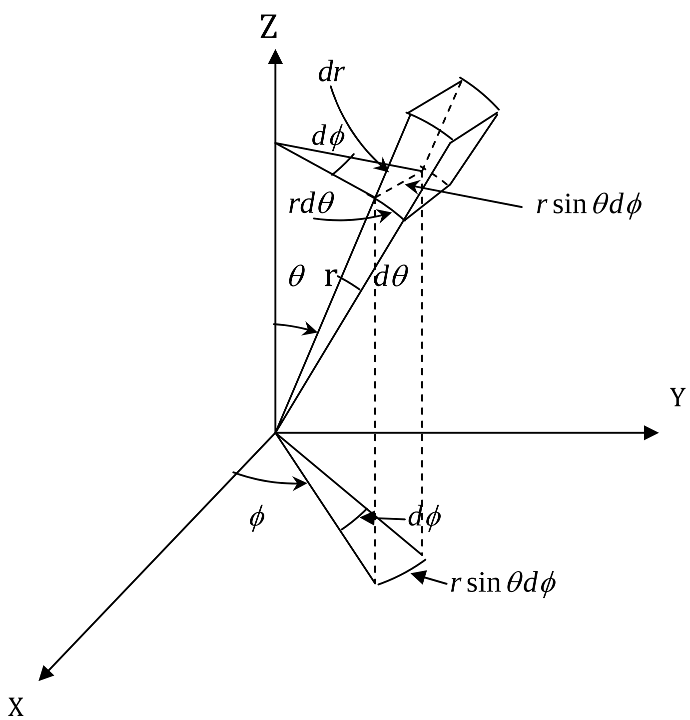
Clearly, $I_X = I_Y$ and $I_Z = I_X + I_Y$ (perpendicular axis theorem)

Now, $I_X = I_Y$ = moment of inertia of a solid sphere having radius R_E about an axis passing through its centre + moment of inertia of an annular ring about an axis passing through its centre & lying in its plane.

I_Z = moment of inertia of a solid sphere having radius R_E about an axis passing through its centre + moment of inertia of an annular ring about an axis passing through its centre & lying perpendicular to its plane.

[$R_E \approx 6.4 \times 10^6$ m]

a. MOMENT OF INERTIA OF A SOLID SPHERE :



SPHERICAL POLAR COORDINATE *fig* 8

Infinitesimal volume element, $d\tau = r^2 \sin\theta d\theta d\phi dr$

For a sphere $I_X{}^{(S)} = I_Y{}^{(S)} = I_Z{}^{(S)}$ [here superscript (s) denotes perfect sphere]

$$I_Z{}^{(S)} = \iiint (x^2 + y^2) dM$$

$$= \iiint (r^2 \sin^2\theta \cos^2\phi + r^2 \sin^2\theta \sin^2\phi)\rho d\tau \qquad [\rho = mass\ density]$$

$$= \rho \iiint r^2 \sin^2\theta (r^2 \sin\theta d\theta d\phi dr) \qquad \text{[assuming } \rho \text{ as constant]}$$

$$= \rho \int_0^{R_E} r^4 dr \int_0^{\pi} \sin^3\theta d\theta \int_0^{2\pi} d\phi$$

$$= \rho \frac{R_E^5}{5} \times 2\pi \times \frac{1}{4} \int_0^{\pi} 4\sin^3\theta d\theta$$

$$= \frac{1}{10} \pi\rho R_E^5 \int_0^{\pi} (3\sin\theta - \sin 3\theta) d\theta$$

$$= \frac{1}{10} \pi\rho R_E^5 [-3\cos\theta + \frac{\cos 3\theta}{3}]_0^{\pi}$$

$$= \frac{1}{10} \pi\rho R_E^5 [-3(\cos\pi - \cos 0) + \frac{1}{3}(\cos 3\pi - \cos 0)]$$

$$= \frac{1}{10} \pi\rho R_E^5 [6 + \frac{1}{3}(-2)]$$

$$= \frac{1}{10} \pi\rho R_E^5 \frac{16}{3}$$

$$= (\frac{4}{3} \pi R_E^3 \rho) \frac{2}{5} R_E^2$$

$$= \frac{2}{5} M_E R_E^2 \qquad [M_E = \frac{4}{3} \pi R_E^3 \rho]$$

$$So, \boxed{I_X{}^{(S)} = I_Y{}^{(S)} = I_Z{}^{(S)} = \frac{2}{5} M_E R_E^2}$$

b. MOMENT OF INERTIA OF AN ANNULAR RING HAVING WIDTH 'h' :

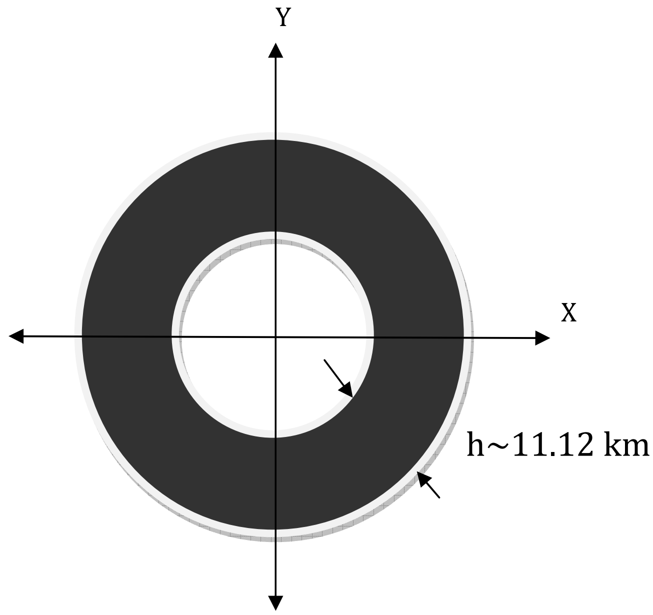


fig 9

According to perpendicular axis theorem, $I_Z = I_X + I_Y = 2I_X = 2I_Y$

Let me consider a ring within r and ($r + dr$).

Moment of inertia of this ring about Z axis,

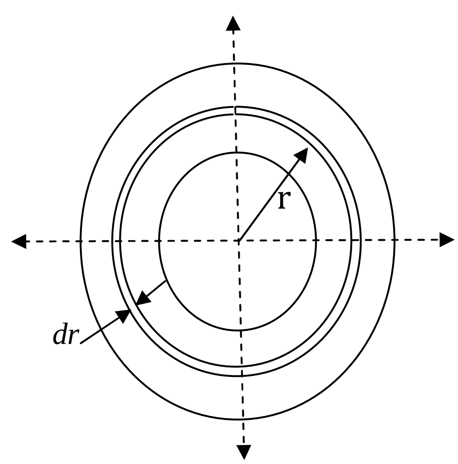


fig 10

$$dI_Z{}^{(R)} = r^2 dM$$

$$= r^2(\sigma 2\pi r dr)$$

$$= 2\pi\sigma r^3 dr$$

$$I_Z{}^{(R)} = 2\pi\sigma \int_{R_E}^{(R_E+h)} r^3 dr$$

$$= 2\pi\sigma \frac{1}{4}[(R_E + h)^4 - R_E{}^4]$$

$$= 2\pi\sigma \frac{1}{4} R_E^4 [(1+\frac{h}{R_E})^4 - 1]$$

$$\approx 2\pi\sigma \frac{1}{4} R_E^4 [1+\frac{4h}{R_E} - 1] \qquad [as \frac{h}{R_E} << 1]$$

$$= 2\pi\sigma R_E{}^3 h$$

$$\approx M_R R_E^2 \qquad [M_R = mass\ of\ the\ annular\ ring]$$

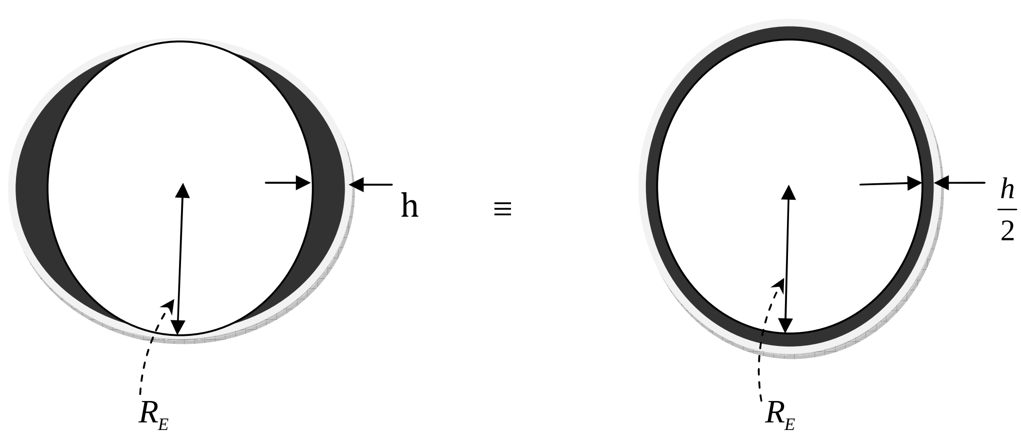
So, $\boxed{I_Z{}^{(R)} \approx M_R R_E{}^2}$

And $\boxed{I_X{}^{(R)} = I_Y{}^{(R)} = \frac{M_R R_E{}^2}{2}}$

Now,

M_R = Mass of the annular ring having h (<< R_E) = mass of the extra portion of the earth i.e. excluding of perfect sphere

To calculate the mass of this extra portion I will approximate it by a spherical shell having width $\frac{h+0}{2} = \frac{h}{2}$ *(=average width as equatorial region has width h and polar region doesn't have any extra width) i.e.*



$\boxed{fig\,11}$

So, I can write

$$M_R = \rho\frac{4}{3}\pi[(R_E + \frac{h}{2})^3 - R_E{}^3]$$

$$= \frac{4}{3}\pi\rho R_E{}^3[(1+\frac{h}{2R_E})^3 - 1]$$

$$\approx \frac{4}{3}\pi\rho R_E{}^3[1+\frac{3h}{2R_E} - 1] \qquad [as\, h << R_E]$$

$$= \rho 2\pi R_E{}^2 h$$

$$= \frac{M_E}{\frac{4}{3}\pi R_E{}^3} 2\pi R_E{}^2 h$$

$$= \frac{3M_E h}{2R_E}$$

So, $\boxed{M_R \approx \frac{3M_E h}{2R_E}} \Leftarrow$ mass of the extra portion

Therefore,

$$I_Z = I_Z{}^{(S)} + I_Z{}^{(R)}$$

$$\approx \frac{2}{5}M_E R_E{}^2 + M_R R_E{}^2$$

$$\approx \frac{2}{5}M_E R_E{}^2 + \frac{3M_E h}{2R_E} R_E{}^2$$

$$= \frac{2}{5}M_E R_E{}^2 + \frac{3}{2}M_E h R_E$$

And,

$$I_Y = I_X = I_X^{(S)} + I_X^{(R)}$$

$$\approx \frac{2}{5}M_E R_E{}^2 + \frac{1}{2}M_R R_E{}^2$$

$$\approx \frac{2}{5}M_E R_E{}^2 + \frac{1}{2}\frac{3M_E h}{2R_E} R_E{}^2$$

$$= \frac{2}{5}M_E R_E{}^2 + \frac{3}{4}M_E h R_E$$

Therefore,

$$\boxed{I_Z \approx \frac{2}{5}M_E R_E{}^2 + \frac{3}{2}M_E h R_E}$$

And, $\boxed{I_Y = I_X \approx \frac{2}{5} M_E R_E^{\ 2} + \frac{3}{4} M_E h R_E}$

NOTE: *Clearly throughout the calculation I have assumed the earth has uniform mass density, but actually mass density of the earth is not uniform. So, here what I obtained is an approximate one.*

- *GRAVITATIONAL POTENTIAL DUE TO A NEARLY SPHERICAL BODY*

To calculate the rate of the 'Precession of the Equinoxes', a slight excursion into potential theory is needed to find the mutual gravitational potential energy of a mass point (representing the sun or the moon) & a non-spherical distribution (representing the earth) of matter.

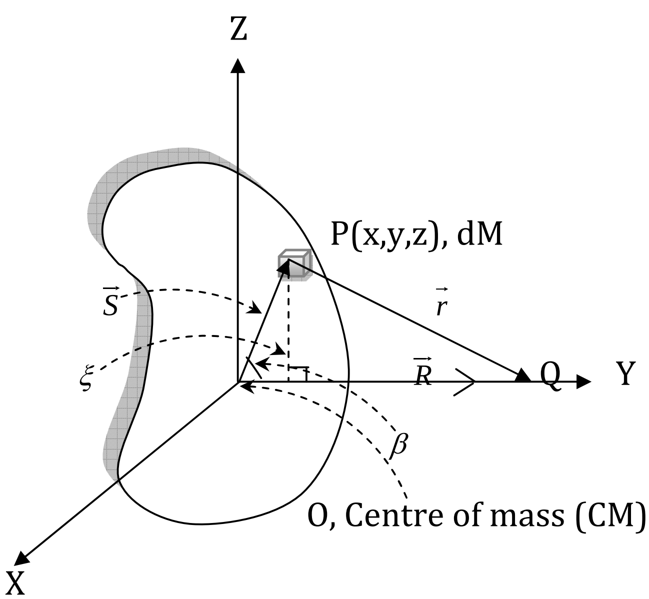


fig 11

Let me consider an arbitrary shaped body having uniform mass distribution with mass M. Also let me consider an infinitesimal mass dM at P(x,y,z). So,

$\vec{S} = x\hat{x} + y\hat{y} + z\hat{z}$

$$\Rightarrow \left|\vec{S}\right| = \sqrt{(x^2 + y^2 + z^{2)}}$$

$$\int dM = M$$

& $\hat{S}.\hat{y} = \cos\beta$

Let, Q be a point on the Y axis. So, the gravitational potential at Q due to this mass distribution is

$$V = -G\int \frac{dM}{r}$$ [r be the distance between P and Q]

Let, R be the distance between centre of mass (CM) & point Q and $\vec{S} + \vec{r} = \vec{R}$

So, from the law of vector addition,

$$r^2 = R^2 + S^2 - 2\vec{R}.\vec{S}$$

$$= R^2 + S^2 - 2RS\,\hat{S}.\hat{y}$$
$$= R^2 + S^2 - 2RS\cos\beta$$

So, $r = \sqrt{R^2 + S^2 - 2RS\cos\beta}$

Therefore,

$$V = -G\int \frac{dM}{r}$$

$$= -G\int \frac{dM}{\sqrt{R^2 + S^2 - 2RS\cos\beta}}$$

$$= -\frac{G}{R}\int \frac{dM}{\sqrt{1 + (S/R)^2 - 2(S/R)\cos\beta}}$$

$$= -\frac{G}{R}\int dM \sum_{n=0}^{\infty} (S/R)^n P_n(\cos\beta)$$ [$P_n(\cos\beta)$ *is Legendre's Polynomial*[7]]

$$= -\frac{G}{R}\int dM - \frac{G}{R}\int dM\,(S/R)\cos\beta - \frac{G}{R}\int dM\,(S/R)^2 \frac{1}{2}(3\cos^2\beta - 1)$$
$$-\textit{Higher order terms of } (S/R)[\rightarrow 0 \textit{ as } S << R]$$

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(7) $P_0(x) = 1$ $P_1(x) = x$

$P_2(x) = \frac{1}{2}(3\cos^2 x - x)$ $P_3(x) = \frac{1}{2}(5x^3 - 3x)$ so on…

$$\approx -\frac{GM}{R} - \frac{G}{R^2}\int S\cos\beta dM - \frac{3G}{2R^3}\int S^2\cos^2\beta dM + \frac{G}{R^3}\int S^2 dM$$
$$= -\frac{GM}{R} - \frac{G}{R^2}\int S\cos\beta dM - \frac{3G}{2R^3}\int S^2(1-\sin^2\beta) dM + \frac{G}{R^3}\int S^2 dM$$
$$= -\frac{GM}{R} - \frac{G}{R^2}\int S\cos\beta dM - \frac{3G}{2R^3}\int S^2 dM + \frac{3G}{2R^3}\int S^2\sin^2\beta dM + \frac{G}{R^3}\int S^2 dM$$
$$= -\frac{GM}{R} - \frac{G}{R^2}\int S\cos\beta dM - \frac{G}{2R^3}\int 2S^2 dM + \frac{3G}{2R^3}\int S^2\sin^2\beta dM$$

For earth, $M = M_E$

So, $$\boxed{V_E \approx -\frac{GM_E}{R} - \frac{G}{R^2}\int S\cos\beta dM_E - \frac{G}{2R^3}\int 2S^2 dM_E + \frac{3G}{2R^3}\int S^2\sin^2\beta dM_E}$$

Here all these terms carry important physical significance, let me explain.

$-\frac{GM_E}{R} \Rightarrow$ is the potential of a point mass, or of a perfect sphere (equivalent to a point mass M at its centre). It thus represents the potential due to earth at an external point at distance R if it were a perfect sphere. It is also the dominant term as r increases, i.e. as we go further away from the earth –in fact, from a very large distance the earth does look like a spherical point mass.

$-\frac{G}{R^2}\int S\cos\beta dM_E = -\frac{G}{R^2}M_E\overline{Y}$ [$\overline{Y}$ is the Y component of the position vector of CM respect to a chosen origin.]

As I have chosen CM at the origin, so this term equals to zero. i.e. this represents sum of all the moments of dM's about CM, which equals to zero.

$-\frac{G}{2R^3}\int 2S^2 dM_E \Rightarrow \int 2S^2 dM_E$

$$= \int 2(x^2+y^2+z^2)dM_E$$
$$= \int 2(x_1^2+x_2^2+x_3^2)dM_E$$ [x_1, x_2, x_3 *are principal axis* & *as* $(x^2+y^2+z^2) = (x_1^2+x_2^2+x_3^2)$]

$$= \int (x_1^2 + x_2^2) dM_E + \int (x_2^2 + x_3^2) dM_E + \int (x_3^2 + x_1^2) dM_E$$

$$= I_3 + I_1 + I_2$$

Where,
I_1 = principal moment of inertia of the earth about x_1 axis
I_2 = principal moment of inertia of the earth about x_2 axis
I_3 = principal moment of inertia of the earth about x_3 axis

4th term,

$$\frac{3G}{2R^3} \int S^2 \sin^2 \beta dM_E$$

$$= \frac{3G}{2R^3} \int \xi^2 dM_E$$ [ξ is the perpendicular distance from P to OQ]

$$= \frac{3G}{2R^3} I_{OQ}$$ [I_{OQ} is moment of inertia of the body about OQ]

So, $$\boxed{V \approx -\frac{GM_E}{R} - \frac{G}{2R^3}[(I_1 + I_2 + I_3) - 3I_{OQ}]}$$

NOTE: *If the earth were a perfect sphere, then* $I_1 = I_2 = I_3 = I_{OQ}$; *so the 2nd term would be zero. Thus the 2nd term is the contribution to the potential from the ellipticity of the earth.*

For the earth $I_1 = I_2 \neq I_3$ i.e. axis of symmetry is along X_3 axis.

$$\boxed{\begin{aligned} I_1 = I_2 &\approx \frac{2}{5} M_E R_E^2 + \frac{3}{4} M_E R_E h \\ I_3 &\approx \frac{2}{5} M_E R_E^2 + \frac{3}{2} M_E R_E h \end{aligned}}$$

Let, l, m, n be the direction cosines of the OQ axis respect to the principal set of axes X_1, X_2, X_3. Then the moment of inertia respect to OQ axis,

$$I_{OQ} = l^2 I_1 + m^2 I_2 + n^2 I_3$$ [*cross terms are absent as these are princcipal set of axes*]

$$= I_1(l^2 + m^2) + I_3 n^2$$

Therefore,

$$(I_1 + I_2 + I_3) - 3I_{OQ}$$

$$= 2I_1 + I_3 - 3I_1(l^2 + m^2) - 3I_3 n^2$$

$$= I_1[2 - 3(l^2 + m^2)] + I_3[1 - 3n^2]$$
$$= I_1[2 - 3(1 - n^2)] + I_3[1 - 3n^2] \qquad [\text{using } l^2 + m^2 + n^2 = 1]$$
$$= I_1[2 - 3 + 3n^2] + I_3[1 - 3n^2]$$
$$= [1 - 3n^2][I_3 - I_1]$$

So, $$\boxed{V \approx -\frac{GM_E}{R} - \frac{G}{2R^3}[1 - 3n^2][I_3 - I_1]}$$

$$= -\frac{GM_E}{R} + \frac{G}{R^3}[I_3 - I_1][\frac{1}{2}(3n^2 - 1)]$$

$$\boxed{V \approx -\frac{GM_E}{R} + \frac{G}{R^3}[I_3 - I_1]P_2(n)}$$

***NOTE:** Here, one interesting thing I should mention that here expansion of **V** is the gravitational analog of the multiple expansion of the electrostatic potential of an arbitrary charged body. Here automatically n=1 term [i.e. $P_1(n)$] doesn't come! Which tells us that there is only one sign of "gravitational charge" and so there can be no" gravitational dipole moment" .*

Further, the inertia tensor is defined analogously to the quadrupole moment tensor. Therefore, the mechanical effects we are seeking can be said to arise from the gravitational quadrupole moment of the oblate earth.

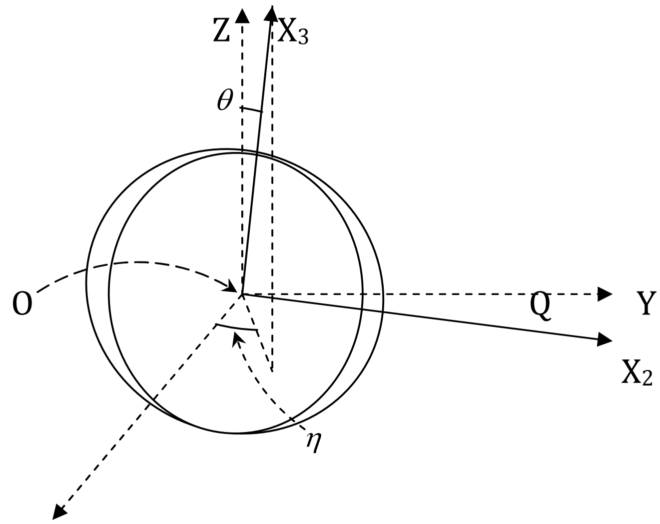
Among the two terms of the potential, 2nd one only depends upon the orientation of the body, & thus could give rise to torque. Let me denote it by V_2, i.e. $\boxed{V_2 = \frac{G}{R^3}[I_3 - I_1]P_2(n)}$

Now, n is the direction cosine of OQ (with X_3 axes).

Here, $X_1, X_2, X_3 \Rightarrow$ *BODY FIXED AXIS*

$X, Y, Z \quad \Rightarrow$ *SPACE FIXED AXIS*

Let, sun or moon is fixed at Q on Y axis. So, n is the direction cosine between the figure axes of the earth (X_3 axes) & the radius vector from the earth's centre to the sun or moon.



[Q ⇒ *sun or moon*]

fig 12

So, n=sin θ cos η

$$\boxed{V_2 = \frac{G}{R^3}[I_3 - I_1]\frac{1}{2}[3\sin^2\theta\cos^2\eta - 1]}$$

Now, according to the observation the orbital period is very rapid compared to the precessional motion and for the purpose of obtaining the mean precession rate it is adequate to average V_2 over a complete orbital period. R can be assumed as constant & the only variation is in $\cos^2\eta$ as η changes continuously with fixed θ. Therefore,

$$\langle V_2 \rangle = \frac{G}{R^3}[I_3 - I_1]\frac{1}{2}[3\sin^2\theta\langle\cos^2\eta\rangle - 1]$$

$$= \frac{G}{R^3}[I_3 - I_1]\frac{1}{2}[3\sin^2\theta(\frac{1}{2}) - 1] \qquad [as\langle\cos^2\eta\rangle_{over\ a\ period} = \frac{1}{2}]$$

$$= \frac{G}{R^3}[I_3 - I_1]\frac{1}{2}[\frac{3}{2}(1 - \cos^2\theta) - 1]$$

[θ be the angle between the normal to the equitorial plane & normal to the ecliptic plane]

$$= \frac{G}{2R^3}[I_3 - I_1][\frac{3}{2} - \frac{3}{2}\cos^2\theta - 1]$$

$$= -\frac{G}{2R^3}[I_3 - I_1][\frac{1}{2}(3\cos^2\theta - 1)]$$

So, $$\boxed{\langle V_2 \rangle = -\frac{G}{2R^3}[I_3 - I_1]P_2(\cos\theta)}$$

Let,

$\dot{\psi}$ = angular velocity of the earth due to rotation about its own figure axis (X_3)

$\dot{\phi}$ = angular velocity of the figure axis (X_3) about space fixed axes Z i.e. it denotes the rate of the precession of the figure axis

$\dot{\theta}$ = angular velocity of the earth due to nutation or wobbling up & down of figure axis (X_3) about space fixed axis Z

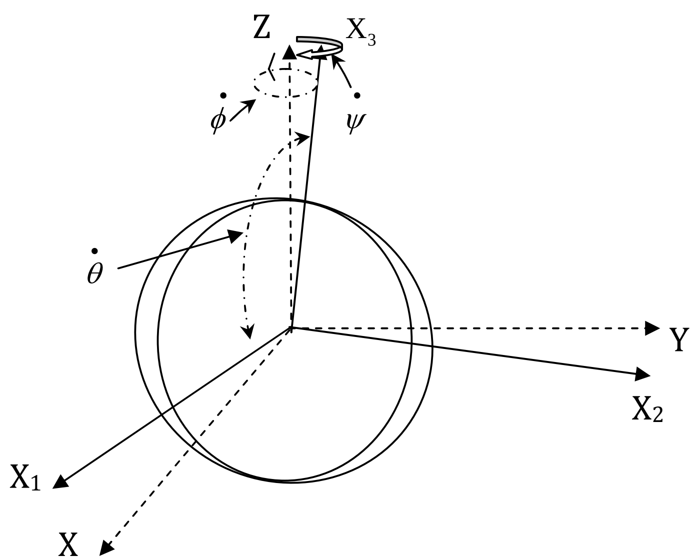


fig 13

Kinetic energy,

$$T=\frac{1}{2}I_1\omega_1^2+\frac{1}{2}I_2\omega_2^2+\frac{1}{2}I_3\omega_3^2$$

$$=\frac{1}{2}I_1(\omega_1^2+\omega_2^2)+\frac{1}{2}I_3\omega_3^2$$

$$=\frac{1}{2}I_1(\dot{\theta}^2+\dot{\phi}^2\sin^2\theta)+\frac{1}{2}I_3(\dot{\psi}+\dot{\phi}\cos\theta)^2$$

So, Lagrangian of this is,

$L=T-\langle U_2\rangle$ [U_2 is potential energy corresponding to V_2]

$$= \frac{1}{2} I_1 (\dot{\theta}^2 + \dot{\phi}^2 \sin^2\theta) + \frac{1}{2} I_3 (\dot{\psi} + \dot{\phi}\cos\theta)^2 - \langle U_2 \rangle$$

So, the equation of motion corresponding to φ is

$$\frac{\partial L}{\partial \theta} - \frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} = 0$$

$$\Rightarrow I_1 \dot{\phi}^2 \sin\theta\cos\theta + I_3(\dot{\psi} + \dot{\phi}\cos\theta)(-\dot{\phi}\sin\theta) - \frac{\partial\langle U_2\rangle}{\partial\theta} = \frac{d}{dt}(I_1\dot{\theta})$$

[we can ignore wobbling i.e. $\dot{\theta} = 0 = \ddot{\theta}$]

$$\Rightarrow I_1 \dot{\phi}^2 \sin\theta\cos\theta - I_3\dot{\phi}\omega_3\sin\theta = \frac{\partial\langle U_2\rangle}{\partial\theta}$$

$$\Rightarrow I_1 \frac{\dot{\phi}^2}{\omega_3}\sin\theta\cos\theta - I_3\dot{\phi}\sin\theta = \frac{1}{\omega_3}\frac{\partial\langle U_2\rangle}{\partial\theta}$$

$$\Rightarrow -I_3\dot{\phi}\sin\theta = \frac{1}{\omega_3}\frac{\partial\langle U_2\rangle}{\partial\theta} \qquad [as\ \omega_3 >> \dot{\phi},\ \text{so}\ \frac{\dot{\phi}}{\omega_3} \approx 0]$$

$$\Rightarrow -I_3\dot{\phi}\sin\theta = \frac{1}{\omega_3}[-\frac{GM_O}{4R^3}(I_3 - I_1)(-6\sin\theta\cos\theta)$$

[$M_O \equiv mass$ of the other object i.e. sun or moon]

$$\Rightarrow \dot{\phi} = -\frac{3GM_0}{2R^3\omega_3}(\frac{I_3 - I_1}{I_3})\cos\theta$$

So, the rate of the precession of the earth's figure axis i.e. rate of the 'Precession of the Equinoxes' is

$$\boxed{\dot{\phi}_{TOTAL} = -\frac{3G}{2\omega_3}(\frac{I_3 - I_1}{I_3})\cos\theta[\frac{M_\odot}{R_{ES}{}^3} + \frac{M_M}{R_{EM}{}^3}]}$$

Where,
$M_\odot$ = mass of the sun
M_M = *mass* of the moon
R_{ES} = distance between the earth & sun
R_{EM} = distance between the earth & moon

NOTE: One thing is that here I have used $\cos\theta$ for case of sun as well as for moon, but actually angle between the normal to the lunar orbit and the spin axis of the earth is not equal to θ. As the mean inclination of the lunar orbit to the ecliptic plane is small ($\sim 5.145^\circ$), so its effect is negligible.

Now,

$$G = 6.67\times10^{-11}\ Nm^2kg^{-2}$$

$$\omega_3 = \frac{2\pi}{24\times60\times60}rad\ s^{-1} = 7.27\times10^{-5}\ rad\ s^{-1}$$

$$I_3 = M_E R_E{}^2(\frac{2}{5}+\frac{3}{2}\frac{h}{R_E})$$

$$I_1 = M_E R_E{}^2(\frac{2}{5}+\frac{3}{4}\frac{h}{R_E})$$

$$\frac{I_3-I_1}{I_3} = \frac{\frac{3}{2}\frac{h}{R_E}-\frac{3}{4}\frac{h}{R_E}}{\frac{2}{5}+\frac{3}{4}\frac{h}{R_E}} = \frac{\frac{3}{4}\frac{h}{R_E}}{\frac{2}{5}+\frac{3}{4}\frac{h}{R_E}} = \frac{1}{\frac{8}{15}\frac{R_E}{h}+2} = \frac{1}{\frac{8}{15}\times\frac{6.4\times10^6}{11.12\times10^3}+2}$$

$$\approx 3.24\times10^{-3}$$

$$\cos\theta = \cos 23^\circ 27'$$

$$M_\odot = 2\times10^{30}\ kg$$

$$R_{ES} = 149.6\times10^9\ m$$

$$M_M = 7.38\times10^{22}\ kg$$

$$R_{EM} = 384.4\times10^6\ m$$

$$\frac{M_\odot}{R_{ES}{}^3} = \frac{2\times10^{30}}{(149.6\times10^9)^3}kg\ m^{-3} = 5.97\times10^{-4}\ kg\ m^{-3}$$

$$\frac{M_M}{R_{EM}{}^3} = \frac{7.38\times10^{22}}{(384.4\times10^6)^3}kg\ m^{-3} = 1.30\times10^{-3}\ kg\ m^{-3}$$

$$\left|\dot{\phi}_{TOTAL}\right| \approx \frac{3\times(6.67\times10^{-11})}{2\times(7.27\times10^{-5})}\times3.24\times10^{-3}\times\cos 23^\circ 27'[(5.97\times10^{-4})+(1.30\times10^{-3})]rad\ s^{-1}$$

$$= 7.76\times10^{-12}\ rad\ s^{-1}$$

$$Therefore, to\ complete\ 1\ complete\ revolution\ it\ will\ take\ \frac{2\pi}{7.76\times10^{-12}}\sec$$

$$= \frac{2\pi}{7.76\times10^{-12}\times(365\times24\times3600)}\ years$$

$$\approx 25675\ years$$

Therefore, ***the rate of the precession of the equinoxes is ~25675 years.***

C. **PRECESSION OF THE BINARY PULSARS**:

Before going to the main part of the calculation I want to give a brief description of the pulsars containing creation of the pulsars, basic properties of the pulsars etc.

- U *BIRTH OF THE STARS*

A star begins its life with the gravitational collapse of a cloud of interstellar gas consisting mostly of 'H' & 'He' that is momentarily cooler, denser, or lower its kinetic energy than its surroundings. Compressional heating raises the core temperature high enough to ignite the thermonuclear reactions. At a temperature of about 10^7 K, a nuclear reaction begins converting hydrogen into the next heavier element, helium, and releasing a large quantity of electromagnetic energy. This energy produces a thermal pressure which tries to expand this gravitational system.

Eventually, however, a significant fraction of H in the star's core is exhausted & there is no longer enough thermonuclear fuel to create the thermal pressure to balance self-gravitational attracting force responsible for gravitational collapse. So again gravitational contraction resumes. Again, heat generated due to this compression raises the temperature until the reactions which burn He to make other elements ignite. So eventually a significant amount of He will be exhausted, the core will again contract, and a new stage of thermonuclear burning will be initiated.

In this way produced thermal pressure from nuclear fusion reaction balances gravitational attraction. Due to these countervailing forces the system is in '*near equilibrium'.* This gravitational system is called 'STAR'.

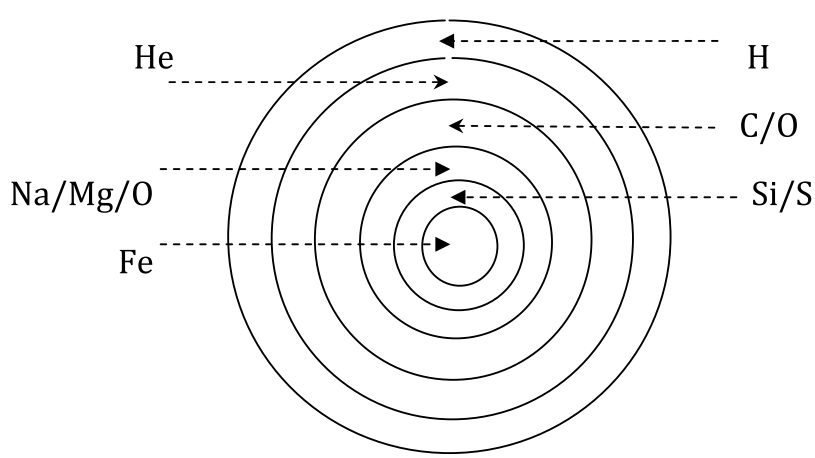
- *DEATH OFTHE STARS*

Two most important properties governing ordinary stars:

a) Ordinary stars are self-gravitating system and so have to be hot inside to sustain thermal pressure that resists the inward pull of gravity.
b) Space, outside of the star is comparatively dark and cold. Therefore, according to the laws of thermodynamics heat flows continuously from the star to the outside region. So the pressure due to heat generated from nuclear fusion reaction decreases. Therefore, due to self-gravity again it will contract which generates pressure. So pressure does increase. The star gets hotter & hotter.

In this never-ending struggle against gravity and the 2nd law of thermodynamics, nuclear energy sources can only provide the temporary respites.

As the time goes on, the star burns hotter and hotter, & so ignites heavier elements which accumulate in the core. Eventually electromagnetic pressure becomes less and less effective against gravitational collapse because when the core has reached the carbon rich phase, the temperature is still insufficient to fuse carbon into iron. Even if a star has reached at sufficient temperature to create iron, no other nuclear fusion reactions producing heavier elements are exothermic and the star has exhausted its nuclear fuel. So when the nuclear stores of energy have run out, the star faces an inevitable end to its brilliant career. The star collapses due to self-gravitational force.



DISTRIBUTION OF MATTER OF A PRE-SUPERNOVA STAR

In the case of big star, this collapse leads to a supernova explosion, while the collapse of smaller stars is much less violent. So, death may come by a violent explosion or it may come more quietly, in a lingering slide toward darkness. But violent or lingering, death is as inevitable for the stars as for us.

In the war between self-gravity and thermodynamics, what ending lie ahead for stars? Astronomers believe four possibilities -

FOUR POSSIBILITIES
1. Nothing may be left; if the explosion is sufficiently violent, all the matter may be dispersed effectively into interstellar space. This would represent an ultimate victory of 'Thermodynamics'.
2. 'Black hole' may be created. This would represent the ultimate victory of self-gravity.
3. A degenerate 'White Dwarf' may be left.
4. 'Neutron star' may be created. 'PULSARS' are actually rapidly spinning magnetized neutron stars having proper magnetic dipole orientation.

REMNANT	***PROGENIT-OR MASS***	***REMNANT MASS***	***SIZE***	***DENSITY***	***MEANS OF SUPPORT***
WHITE DWARF	$M_* < 8M_\odot$	$M_{WD} < 1.4M_\odot$ [8]	$R_{WD} \sim R_{EARTH}$	$1 ton/cm^3$	e^{-1} degeneracy [9]
NEUTRON STAR	$8M_\odot < M_* < 20M_\odot$	$1.4M_\odot \le M_{NS} < 3M_\odot$	$R_{NS} \sim 10\,km$	200 $milliton/cm^3$	$'n'$ degeneracy
BLACK HOLE	$M_* > 20M_\odot$	$M_{BH} > 3M_\odot$	$R_{BH} \sim 0$ $R_{SCH} = \frac{2GM}{C^2}$	∞	*none*

As 'Pulsars' are believed to be the rapidly rotating magnetized neutron stars having some other specific properties, so let me first describe 'Neutron Stars'.

---

[8] $1.4M_\odot = M_{CHANDRASEKHAR}$ .Subrahmanyan Chandrasekhar first (1930) calculated the max mass ($1.4M_\odot$) for a white dwarf.

[9] I will describe later what is meant by 'degeneracy'.

- NEUTRON STARS $\Rightarrow$ PULSARS

When the nuclear fuel inside the star is reached in iron state, the star is no longer able to produce energy in the core via further nuclear burning process as iron is the most tightly bound nucleus. Nuclear reactions will continue, however, because of the extremely high temperatures in the massive star's core. These further reactions have a devastating effect on the star, because they take energy out of the core. At such high temperatures and densities the gamma-ray photons present in the core have sufficient energy to destroy the heavy nuclei produced in the many stages of nuclear reactions, e.g. :

$$\gamma + {}^{56}Fe \rightarrow 13\,{}^{4}He + 4n$$

This process, called *photodisintegration* undoes the work of a stellar lifetime in the core and removes the thermal energy necessary to provide pressure support. The result is a catastrophic collapse of the core, which cannot be halted until the core has shrunk to a size of about 10 km and a density of the order of 200milliontons/cm$^{3.}$ Under such extreme conditions electron degeneracy cannot support the stellar core, and the free electrons are forced together with protons to form neutrons:

$$p^{+} + e^{-} \rightarrow n + \nu_{e}$$

The neutrinos, which escape directly from the core, result in further energy loss and even faster collapse. When the core has collapsed down to a size of about 10 km, *neutron degeneracy* sets in causing the core to stiffen and this neutron degeneracy pressure can balance the inward gravitational pull. This newly formed stable astronomical object is named as '*Neutron Star'* as it is mainly formed by 'neutron'.

NOTE: DEGENERATE PRESSURE – Degenerate pressure is a quantum mechanical effect, a consequence of the 'Pauli's Exclusion Principle' and 'Heisenberg's Uncertainty Principle'. According to the exclusion principle *two fermions can't occupy the same quantum state at the same time.* In case of very compact object electrons are squeezed too closely together, they can't be distinguished by their position, and the exclusion principle requires them to have different energy levels. The number of available low energy states is too small and many electrons are forced into higher energy states. Also as they are now very compact, so we can say an electron's position is well defined. Therefore, since $\boxed{\Delta x \Delta p_x \sim \frac{\hbar}{2}}$ (Uncertainty Principle), then we must say that their momentum is extremely uncertain since they are located in a very confined space. So, the electrons must be moving with a *very high velocity on average*. These correspond to a very high pressure named as '*electron degenerate pressure*'. It has the following important properties:

- ❖ Pressure is independent of the temperature, so the pressure doesn't go down as the star cools.
- ❖ Compression does not lead to heating.

$Electron\ \deg enerate\ pressure\ in\ a\ material\ can\ be\ computed\ as$

$$\boxed{P_D^e = \frac{4\pi^2\hbar^2}{20 m_e m_p^{5/3}} (\frac{3}{\pi})^{2/3} (\frac{\rho}{\mu})^{5/3}} \quad [Non-relativistic\ formula]$$

$where,$

$\hbar = Plank's\ cons\ divided\ by\ 2\pi$

$m_e = mass\ of\ the\ electron$

$m_p = mass\ of\ the\ proton$

$\rho = density$

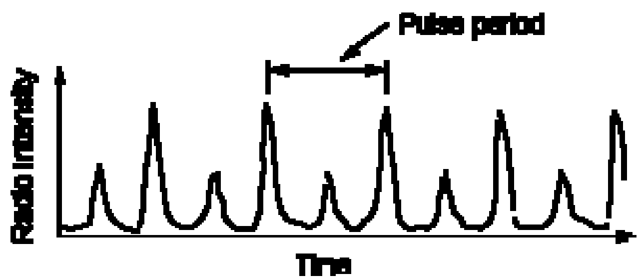
$\mu = proton\ to\ electron\ mass\ ratio$

At common density it is so low that it can be neglected.

It is the force that supports white dwarf against their self-gravity. If the star is more massive, it is possible to absorb the electrons into nucleons, converting proton into neutrons. If this process is going on continuously then the electron degenerate pressure can't able to support against self-gravity. So the star starts to collapse. But now '*neutron degeneracy pressure*' starts to play. This gravitational collapse becomes stop when this pressure becomes sufficient to balance the self-gravity. This force supports neutron stars against their self-gravity.

- *PULSARS*

❖ <u>*DISCOVERY*</u>: As with many things in astronomy, the discovery of the 'Pulsars' came in unexpected fashion. In 1967, Cambridge graduate student Jocelyn Bell and Anthony Hewish were using a special radio telescope to look for radio scintillation, fluctuations in the signal from distant radio sources. On November 28, Bell discovered a source with an exceptionally regular pattern of radio flashes. These radio flashes occurred every 1.33728 seconds. The period was extremely precise, with a variation of no more than 1 part in 10 million. They puzzled by thinking what sort of object could produce such an extremely regular pattern of radio flashes. Some speculated that the sources might actually be beacons set up by a distant extraterrestrial civilization, and so the sources were initially known in some circles as "LGMs", for "Little Green Men". After a few weeks, however, three more rapidly pulsating objects were detected. After that they were named as 'PULSARS'.



❖ _Characteristics of the pulsars :_

1. These are comparatively ***very small astronomical objects (diameter ~10-15 km).***
   The ***sharpness of the radio pulses emitted by the pulsars*** suggests that they must be very small objects, may be about 10-15 km in diameter. If the object were larger, radio waves emitted from more distant regions of the pulsar would arrive after a long time than those emitted from nearer regions, which denotes the spreading of the pulse.

2. Pulsars are actually ***neutron stars***.
   The pulse period (small) indicated that the objects are spinning rapidly. The ***rate of rotation implies an object of stellar mass, as anything lighter will simply tear itself apart due to very strong centrifugal force***. The only thing that can meet such constraints is a neutron star due to its strong self-gravity. But it also has an upper limit of the mass. As a neutron star becomes more massive, it must become stiffer to maintain itself, and the speed of sound through the star increases accordingly. Above three solar masses, the speed of sound exceeds that of light, which is ruled out by Einstein's theory of

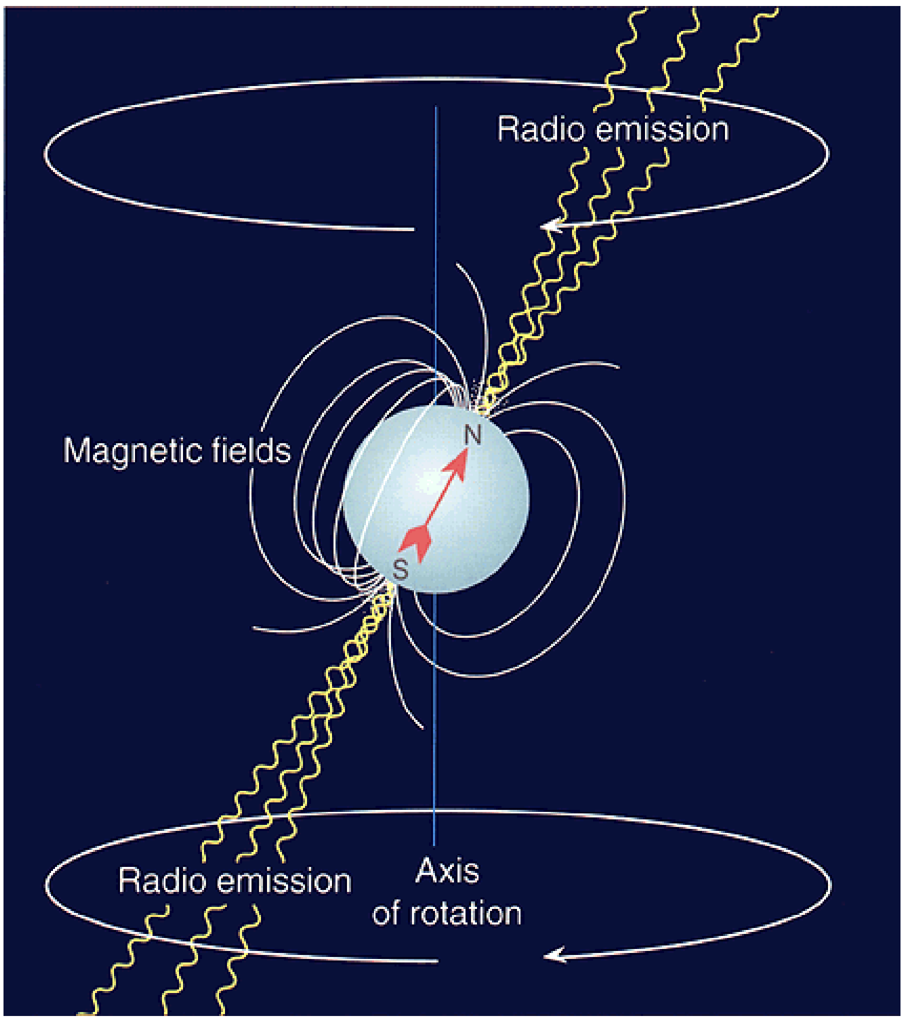
relativity. So it can't have mass greater than three solar mass.

3. Pulsars are ***spinning very rapidly.***
While neutron stars are created from the main star, then volume becomes enormously less than that of original stars. So according to the ***conservation of angular momentum*** spinning rate should increase accordingly. So the spinning rate becomes very high. A star like the sun, rotating once per month would rotate about 1000 times per second when contracted down to 10 km in size.
Astronomers also believe that there may be an effect of the ***off-centre kick*** on the neutron star after supernova explosion.

4. The pulsars are also ***very hot up to a*** 10^5 ***or a million K.***
The ***surface area of the pulsars is very small relative to its mass***, and so the energy trapped during its formation can radiate away only very slowly. So it is very hot.

5. Pulsars have extraordinary ***intense magnetic field (~***10^{12} ***gauss)***. The "pulses" of high-energy radiation we see from a pulsar are due to a misalignment of the pulsar's rotation axis and its magnetic axis. Therefore, the magnetic field is rotating around its spin axis, so according to Maxwell's Electromagnetic Theory it generates an electric field. Due to this immensely strong electric field all charge particles – electrons, positrons etc are experienced a strong electrostatic force, also moving charge particles are experienced magnetic force. Therefore, it sets up an immense flow of electrons, positrons, and ions over its surface and scatters them into space. This flow of charged particles is known as the "***pulsar wind***", analogous to the "Solar wind" emitted by our Sun. Most of the charged particles coming out of the pulsars move out through the two magnetic poles of the pulsars due to a velocity component of the charged particles along the magnetic field. This phenomenon is called '***dipole radiation'.***
Now as charge particles are accelerated so they radiate electromagnetic radiation all along the electromagnetic spectrum corresponding to their frequency. Again when generated gamma ray passes through electric field, pair production can be occurred which again produce radiation by the above process.
Pulsars pulse because the rotation of the neutron star causes the radiation generated within the magnetic field to sweep in and out of our line of sight with a regular period.

But why it has such a strong magnetic field?? One obvious reason is that ***conservation of magnetic flux.*** According to which as the core collapses the magnetic field lines are pulled more closely together, intensifying the magnetic field to values of the order of 10^{12} gauss. Astronomers believe another possible reason which may the microscopic source of high magnetic field. They believe that inside the pulsar ***charged particles may be in superconducting state*** which is responsible for high current density and for which high magnetic field may be created.



A DIAGRAM OF A PULSAR SHOWING ITS ROTATION AXIS, ITS MAGNETIC AXIS, AND ITS MAGNETIC FIELD

6. Due to dipole radiation pulsar continuously loses energy so it gradually slows down. The rate at which the spin of a pulsar slows down indicates its rate of energy emission, and even though the beams are intense they only account for a small fraction of the energy emission of the pulsar. Most of the energy emission is likely in the form of the pulsar wind and other unseen radiation. After about ten million years, the pulsar slows down and no longer has enough energy to emit pulses.

Therefore, a pulsar is a fast-spinning, highly magnetized neutron star, formed (in most cases) in supernova explosion, that sends out regular directional pulses of radiation as it rotates, in the manner of a lighthouse beam; the pulsar effect is seen if the beam happens to sweep in our direction. At first pulsars were found in radio wavelengths but after that X-ray, optical-ray, gamma-ray pulsars have been found. Examples of some pulsars:**PSR-1919+21**, **PSR-B1919+21** and **PSR-J1921+2153.**

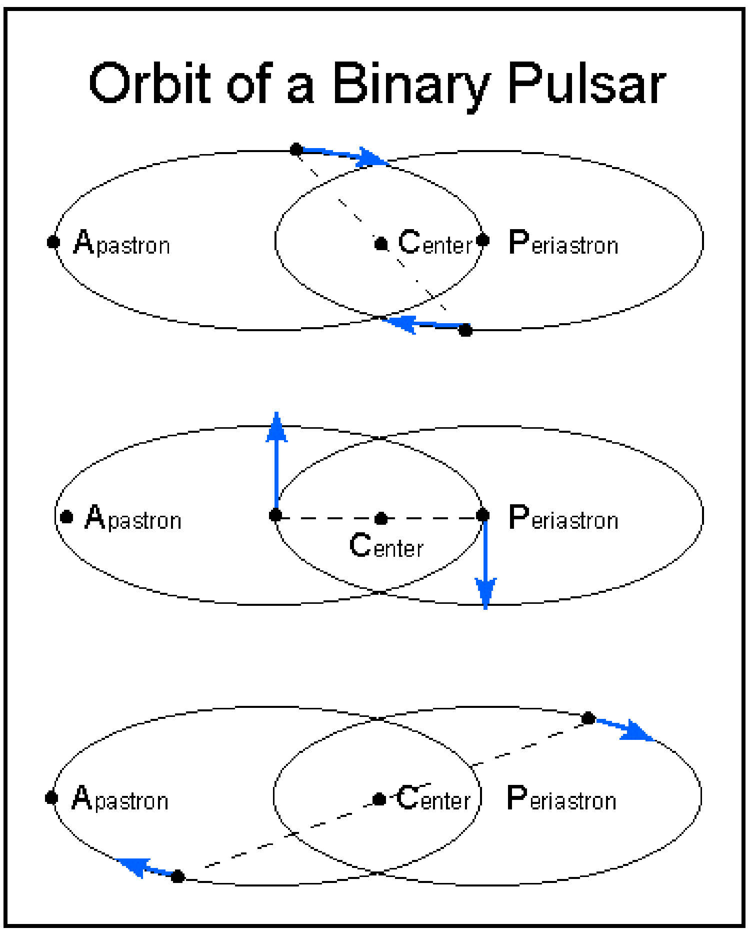
- *PULSAR IN BINARY SYSTEM*

In astronomy binary system means there are two astronomical objects those orbit around a common center of mass.

So, 'binary pulsar' is a pulsar in a binary system i.e. it has another companion star to it, often another pulsar or white dwarf or neutron star. The first binary pulsar, PSR 1913+16 or the "Hulse-Taylor binary pulsar" was discovered in 1974 by Joseph Hooton Taylor and Russell Huls.

In a binary system if *one* of the objects is a *massive compact* star such as neutron star and the *other* is a *normal star*, then neutron star can pull gas & dust off the normal star due to its powerful gravitational field and accrete it onto itself. Since the stars are revolving around each other and since angular momentum must be conserved (ignoring the effect due to other astronomical objects on these), this gas & dust can't fall directly onto neutron star, but instead spirals in to the neutron star. Thus material flowing from the normal star to neutron star piles up in a dense spinning disc orbiting the neutron star. This disc is called '***accretion disc'.*** The material in the disc becomes very hot due to friction and being tugged on by neutron star & eventually loses angular momentum and gradually falls onto the neutron star. Since this hot gas is being accelerated it radiates energy, usually in X-rays. Since the gravitational pull on the material is the basic source of energy for this emission, these pulsars are often called "accretion powered pulsars".

The pulsar and its companion both follow elliptical orbits around their common center of mass. Each star moves in its orbit according to Kepler's Laws; at all times the two stars are found on opposite sides of a line passing through the center of mass. In case of binary pulsar PSR 1913+16 the period of the orbital motion is 7.75 hours, and the stars are believed to be nearly equal in mass, about 1.4 solar masses. The minimum separation at *periastron* is about 1.1 solar radii; the maximum separation at *apastron* is 4.8 solar radii. A star in an elliptical orbit will move slower when it is at apastron than when it is a periastron. In an eccentric orbit such as that of PSR 1913+16, the radial velocity varies from a minimum of 75 km/sec to a maximum of 300 km/sec.



One of the most important features of the discovery of binary pulsar system is that it provides a powerful test of the predictions of the behavior of time perceived by a distant observer according to Einstein's General Theory of Relativity. When they are closer together, near apastron, the gravitational field is stronger, so that the passage of time is slowed down -- the time between pulses (ticks) lengthens just as Einstein predicted. The pulsar clock is slowed down when it is travelling fastest and in the strongest part of the gravitational field; it regains time when it is travelling more slowly and in the weakest part of the field. So it gives us an experimental verification of Einstein's Theory of General Relativity.

The theory of general relativity predicts that masses being accelerated by gravitational field should emit ``*gravitational radiation*'' in the same way that charged particles (like electrons) emit electromagnetic radiation when they are accelerated. So according to this theory binary pulsar system should radiate gravitational waves. Thus system will lose energy gradually as orbital energy is converted into gravitational waves. This leads to the shrinking of the orbit. In 1983, Taylor and collaborators reported that there was a systematic shift in the observed time of periastron relative to that expected if the orbital separation remained constant. This actually confirms the prediction of Einstein.

- *MATHEMATICAL APPROACH & TOOLS*

For further calculation I will follow the 'Schwinger's Source Theory' approach. This theory is basically a theoretical description of particle interaction unlike geometrical viewpoint of the 'Einstein's Theory of General Relativity' based on the assumption of the existence of ***a spin 2 massless particle named as 'graviton' acting as the mediator of the gravitational force*** just like massless spin '1' particle named 'photon' in case of Electromagnetic force, massless spin '1' particle named 'gluon' in strong force and W & Z bosons for electroweak force.

Here why we require massless particle that can be estimated using simple arguments based on the uncertainty principle. The amount of energy required for the exchange of a force mediating particle is found using Einstein's mass-energy equivalence principle as

$$\boxed{\Delta E \approx mc^2}$$

Now how long the particle can exist that can be calculated using uncertainty principle. That is

$$\Delta t \approx \frac{\hbar}{\Delta E} = \frac{\hbar}{mc^2}$$

The special theory of relativity tells us that nothing can travel faster than the speed of light. So, we can use the speed of light to set an upper limit on the velocity of the force-carrying particle, and estimate the range it travels in time Δt, that is,

$$\Delta x = c\Delta t = \frac{c\hbar}{mc^2} = \frac{\hbar}{mc}$$

For $\Delta x \to \infty$ (as gravitational force is an infinitely extended force), m=0. So graviton must have zero rest mass. [This above logic is not applicable for strong force]

Also graviton can't have spin 1 because there is a fundamental difference between electromagnetic force and gravitational force. That is electromagnetic force can be attractive as well as repulsive but gravitational force can only be attractive. So, there should be a characteristics difference between photon and graviton. So, graviton can't have same spin as that of photon. Also graviton can't have spin 0. This reversely can be verified. If we assume it has zero spin then we will see light can't be bended near a heavy object. But it is experimentally verified that light indeed bends near a heavy object. So, graviton can't have zero spin.

This theory is very much analogous to the electromagnetic theory. In this theory there is existence of force & the space-time remains flat unlike Einstein's theory of general relativity where there is nothing called force & space-time can have any curvature. In Einstein's theory mass can change the curvature of the space-time and the bending of light near a heavy object including other phenomena (which we think that are happening due to force acting on the object) are the consequence of the curvature of the space-time. Curvature of the space-time near a gravitational body is proportional to the mass of the body. Therefore, light bends more near a heavy object. This is equivalent to the argument that a massive body exerts more gravitational force on other body including light.

By using this theoretical approach the precession rate of gyroscopes in gravitating system can be calculated which does agree with the experimental observation.

➢ PRECESSION OF THE GYROSCOPES IN GRAVITATING SYSTEM

Suppose there are two bodies having mass m_1 & m_2 respectively and

R_{12} = distance between the centers of the two bodies

$\vec{r}$ = position vector of one (arbitrary) of the constituent particles of the 1st body respect to a chosen origin

$\vec{r}'$ = position vector of one (arbitrary) of the constituent particles of the 2nd body respect to the chosen origin

G = universal gravitational constant

E = gravitational interaction energy

$T_i^{\mu\nu}(\vec{r})$ = total energy-momentum tensor of the 'i'th particle having position vector $\vec{r}$

$T_i(\vec{r})$ = trace of the total energy-momentum tensor of the 'i'th particle having position vector $\vec{r}$

Then according to the source theoretic description of the gravity gravitational interaction energy between the 'i'th particle (of the 1st body and it has position vector $\vec{r}$) and the 'j'th particle (of the 2nd body and it has position vector $\vec{r}'$),

$$E_{ij} = -G\iint \frac{d^{(3)}r\, d^{(3)}r'}{\left|\vec{r}-\vec{r}'\right|}\left[2T_i^{\mu\nu}(\vec{r})T_{j\,\mu\nu}(\vec{r}') - T_i(\vec{r})T_j(\vec{r}')\right] \quad [\mu \,\&\, \nu = 0, 1, 2, 3]$$

Trace, $T = T^{kk} - T^{00}$ [k=1, 2, 3]

The above equation is valid when recoiling effect is negligible.

Due to universal coupling of gravity to all energy-momentum tensor we can't neglect interaction between gravitons themselves. So, $T_{\mu\nu}$ is the *total* energy momentum tensor of the 'i' th particle, i.e.

$$T^{\mu\nu} = T_1^{\mu\nu} + T_2^{\mu\nu} + t_G^{\mu\nu}$$

Where, i denotes the 'i' th particle, 'j' denotes the j th particle and 'G' denotes the gravitational field.

Therefore, the corresponding **total** gravitational interaction energy is

(a) Interaction energy between i th particle & j th particle = $E_{i\,j}$

(b) Interaction energy between i th particle & gravitational field(G)= $E_{i\,G}$

(c) Interaction energy between i th particle & gravitational field(G)= $E_{j\,G}$

Here I'll consider only interaction between i and j i.e. $E_{i\,j}$

Now, for a system of particles $\sum_i m_i$ energy-momentum tensor is given by

$$\boxed{T^{\mu\nu}(\vec{r}) = \sum_i \frac{P_i^{\mu} P_i^{\nu}}{E_i} \delta^{(3)}(\vec{r} - \vec{r}_i(t))}$$

Where, P_i^{μ} = μ th component of the 4-momentum of the i th particle

E_i = total relativistic energy of the i th particle = P_i^0 (i.e. velocity of light in vacuum, c=1)

$\vec{r}_i(t)$ = position vector of the i th particle at time t respect to the chosen origin

$\delta^{(3)}$ = delta function in 3 dimension

And the trace is given by

$$\boxed{T(\vec{r}) = -\sum_i \frac{m_i^2}{E_i} \delta^{(3)}(\vec{r} - \vec{r}_i(t))}$$

Now, as our main assumptions are ***negligible recoiling effect and negligible retardation effect,*** so we must have to choose one of the bodies much more massive than other. So, let me assume $\boxed{m_1 >> m_2}$ such that m_1 can be assumed almost stationary compared to m_2. Also, let me assume m_1 as well as m_2 are point particles and m_1 is situated at the origin of co-ordinate system i.e. $\vec{r}_1(t=0) = 0$ & $\vec{r}_1(t) \approx 0$. Therefore,

$$T_1^{\mu\nu}(\vec{r}) = \frac{P_1^\mu P_1^\nu}{E_1}\delta^{(3)}(\vec{r} - \overrightarrow{r_1(t)})$$

$$= \frac{P_1^\mu P_1^\nu}{E_1}\delta^{(3)}(\vec{r})$$

$$= \frac{P_1^\mu \delta^{\mu 0} P_1^\nu \delta^{\nu 0}}{E_1}\delta^{(3)}(\vec{r}) \qquad [as\ \dot{\vec{r}}_1(t) \approx 0]$$

$$= \frac{(P_1^0)^2 \delta^{\mu 0}\delta^{\nu 0}}{E_1}\delta^{(3)}(\vec{r}) \qquad [P_1^0 = E_1 = \sqrt{m_1^2 + \vec{p}_1^{\,2}} = m_1$$

$$as\ linear\ momentum\ \vec{p}_1 \approx \vec{0}]$$

$$\boxed{T_1^{\mu\nu}(\vec{r}) = m_1 \delta^{\mu 0}\delta^{\nu 0}\delta^{(3)}(\vec{r})}$$

and,

$$T_1(\vec{r}) = T_1^{kk}(\vec{r}) - T_1^{00}(\vec{r})$$

$$= 0 - T_1^{00}(\vec{r})$$

$$\boxed{T_1(\vec{r}) = -m_1\delta^{(3)}(\vec{r})}$$

So, the interaction energy between particles m_1 and 2nd body,

$$E_{12} = -G\iint \frac{d^{(3)}r d^{(3)}r'}{|\vec{r} - \vec{r}'|}[2T_1^{\mu\nu}(\vec{r})T_{2\ \mu\nu}(\vec{r}') - T_1(\vec{r})T_2(\vec{r}')]$$

$$= -G\iint \frac{d^{(3)}r d^{(3)}r'}{|\vec{r} - \vec{r}'|}[2m_1\delta^{\mu 0}\delta^{\nu 0}\delta^{(3)}(\vec{r})T_{2\ \mu\nu}(\vec{r}') - T_1(\vec{r})T_2(\vec{r}')]$$

$$= -G\iint \frac{d^{(3)}r d^{(3)}r'}{|\vec{r} - \vec{r}'|}[2m_1 T_{2\ 00}(\vec{r}') + m_1 T_2(\vec{r}')]\delta^{(3)}(\vec{r})$$

$$= -Gm_1\int_{r'} \frac{d^{(3)}r'}{|\vec{r}'|}[2T_{2\ 00}(\vec{r}') + T_2(\vec{r}')]$$

$$= -Gm_1\int_{r'} \frac{d^{(3)}r'}{|\vec{r}'|}[2T_{2\ 00}(\vec{r}') + T_{2\ kk}(\vec{r}') - T_{2\ 00}(\vec{r}')]$$

$$= -Gm_1\int_{r'} \frac{d^{(3)}r'}{|\vec{r}'|}[T_{2\ 00}(\vec{r}') + T_{2\ kk}(\vec{r}')]$$

$$= -Gm_1\int_{r'} \frac{d^{(3)}r'}{|\vec{r}'|}[T_2^{00}(\vec{r}') + T_2^{kk}(\vec{r}')]$$

$$= -Gm_1 \int_{r'} \frac{d^{(3)}r'}{|\vec{r'}|} [E_2 \delta^{(3)}(\vec{r'} - \vec{r_2}'(t)) + \frac{p_2^2}{E_2} \delta^{(3)}(\vec{r'} - \vec{r_2}'(t))]$$

$$[as\, T_2^{00} = \frac{(p_2^0)^2}{E_2} \delta^{(3)}(\vec{r'} - \vec{r_2}'(t)) = \frac{E_2^2}{E_2} \delta^{(3)}(\vec{r'} - \vec{r_2}'(t)) = E_2 \delta^{(3)}(\vec{r'} - \vec{r_2}'(t))]$$

$$= -Gm_1 \int_{r'} \frac{d^{(3)}r'}{|\vec{r'}|} [E_2 + \frac{p_2^2}{E_2}] \delta^{(3)}(\vec{r'} - \vec{r_2}'(t))$$

$$= -\frac{Gm_1}{|\vec{r_2}'(t)|} [E_2 + \frac{p_2^2}{E_2}]$$

$$= -\frac{Gm_1}{|\vec{r_2}'(t)|} [m_2 (1 + \frac{\vec{p}_2^{\,2}}{m_2^2})^{1/2} + \frac{\vec{p}_2^{\,2}}{m_2} (1 + \frac{\vec{p}_2^{\,2}}{m_2^2})^{-1/2}]$$

$$\approx -\frac{Gm_1}{|\vec{r_2}'(t)|} [m_2 (1 + \frac{\vec{p}_2^{\,2}}{2m_2^2}) + \frac{\vec{p}_2^{\,2}}{m_2} - \frac{\vec{p}_2^{\,4}}{m_2^{\,3}}]$$

$$= -\frac{Gm_1 m_2}{|\vec{r_2}'(t)|} [1 + \frac{3\vec{p}_2^{\,2}}{2m_2^2} + \ldots]$$

$$\boxed{E_{12}^{\,non-recoil} \approx -\frac{Gm_1 m_2}{R_{12}} [1 + \frac{3\vec{p}_2^{\,2}}{2m_2^2} + \ldots]} \qquad [where, |\vec{r_2}'(t)| = R_{12} = \text{distance between the centers of the two}]$$

Here, from the above equation it is clear that this interaction energy is actually different from that of Newtonian gravity.

$$E_{12}^{Newtonian} = -\frac{Gm_1 m_2}{R_{12}}$$

So, here we get some extra terms which are actually responsible for many interesting phenomena including precession of the spin axis of astronomical body.

Now, let me consider ***the general treatment including recoil:***

Previously I got $E_{12}^{\,non-recoil} \approx -\frac{Gm_1 m_2}{R_{12}} [1 + \frac{3\vec{p}_2^{\,2}}{2m_2^2} + \ldots]$ [excluding recoil]

If we take the recoiling effect into account then the extra terms which we will get are due to the motion of m_1. It comes as

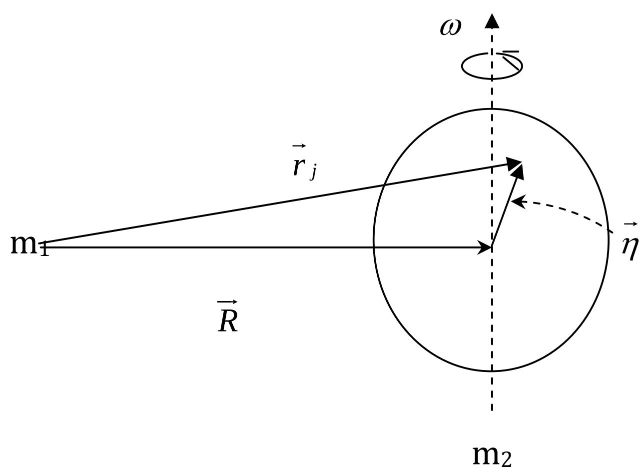
$$E_{12}{}^{recoil} \approx -\frac{Gm_1m_2}{R_{12}}[1+\frac{3}{2}\overrightarrow{v_2}^2+\frac{3}{2}\overrightarrow{v_1}^2-\frac{7}{2}\vec{v}_1.\vec{v}_2-\frac{(\vec{v}_1.\overrightarrow{r_{12}})(\vec{v}_2.\overrightarrow{r_{12}})}{2R^2}+...]$$

Now, suppose ***m_2 is perfectly spherical in shape and spinning respect to its own axis passing through the center.*** The energy-momentum tensor for such a system is

$$T_2{}^{\mu\nu} = \sum_j \frac{P_j^{\mu}P_j^{\nu}}{E_j}\delta^{(3)}(\vec{r}-\vec{r}_j) \qquad [\,j \text{ denotes } j\text{th particle constituting } m_2 \text{ i.e.} \sum_j m_j = m_2\,]$$

$$T_2^{00} = \sum_j E_j \delta^{(3)}(\vec{r}-\vec{r}_j)$$

$$T_2^{kk} = \sum_j \frac{P_j^{k}P_j^{k}}{E_j}\delta^{(3)}(\vec{r}-\vec{r}_j)$$



$$T_2^{kk} = \sum \frac{p_j^2}{E_j}\delta^{(3)}(\vec{r}-\overrightarrow{r_j})$$

$$T_2^{00}+T_2^{kk} = \sum[E_j+\frac{p_j^2}{E_j}]\delta^{(3)}(\vec{r}-\overrightarrow{r_j})$$

$$= \sum[m_j+\frac{3}{2}m_j\overrightarrow{v_j}^2+...]$$

$$\approx \int[\rho(\eta)d^{(3)}\eta+\frac{3}{2}\rho(\eta)d^{(3)}\eta(\vec{v}_{cm}+\vec{\omega}\times\vec{\eta})^2]$$

[*for continuous system*]

$$E_{12} = -Gm_1 \int \frac{d^{(3)}r}{|\vec{r}|}[T_2^{00} + T_2^{kk}]$$

$$= -Gm_1 \int \frac{d^{(3)}r}{|\vec{r}|}[\int [\rho(\eta) d^{(3)}\eta + \frac{3}{2}\rho(\eta) d^{(3)}\eta(\vec{v}_{cm} + \vec{\omega}\times\vec{\eta})^2]\delta^{(3)}(\vec{r} - \vec{r}_j)$$

$[\vec{v}_{cm} = velocity\ of\ the\ center\ of\ mass\ of\ the\ 2nd\ body\ respect\ to\ origin]$

$$= -Gm_1 \int [\rho(\eta) d^{(3)}\eta + \frac{3}{2}\rho(\eta) d^{(3)}\eta(\vec{v}_{cm} + \vec{\omega}\times\vec{\eta})^2]\frac{1}{|\vec{r}_j|}$$

$$= -Gm_1 \int [\rho(\eta) d^{(3)}\eta + \frac{3}{2}\rho(\eta) d^{(3)}\eta(\vec{v}_{cm} + \vec{\omega}\times\vec{\eta})^2]\frac{1}{|\vec{R} + \vec{\eta}|}$$

$$\approx -Gm_1 \int [\rho(\eta) d^{(3)}\eta + \frac{3}{2}\rho(\eta) d^{(3)}\eta(\vec{v}_{cm} + \vec{\omega}\times\vec{\eta})^2][\frac{1}{R} - \frac{\vec{R}.\vec{\eta}}{R^3} - ...]$$

$$\approx -Gm_1 \int [\rho(\eta) d^{(3)}\eta \frac{1}{R} - \rho(\eta) d^{(3)}\eta \frac{\vec{R}.\vec{\eta}}{R^3}] - Gm_1 \int \frac{3}{2}\rho(\eta) d^{(3)}\eta[\vec{v}_{cm}^{\ 2} + 2\vec{v}_{cm}.(\vec{\omega}\times\vec{\eta}) + (\vec{\omega}\times\vec{\eta})^2][\frac{1}{R} - \frac{\vec{R}.\vec{\eta}}{R^3}]$$

$$= -Gm_1[\frac{m_2}{R} - \frac{\vec{R}}{R^3}.\int \vec{\eta} dm_2] - Gm_1 \int [\frac{3}{2}\rho(\eta) d^{(3)}\eta \frac{1}{R} - \frac{3}{2}\rho(\eta) d^{(3)}\eta \frac{\vec{R}.\vec{\eta}}{R^3}][\vec{v}_{cm}^{\ 2} + 2\vec{v}_{cm}.(\vec{\omega}\times\vec{\eta}) + (\vec{\omega}\times\vec{\eta})^2]$$

$$= -\frac{Gm_1m_2}{R} - Gm_1[\frac{3}{2}\frac{1}{R}\vec{v}_{cm}^{\ 2} m_2 - \frac{3}{2}\vec{v}_{cm}^{\ 2}\frac{\vec{R}}{R^3}.\int \vec{\eta} dm_2 + 3\frac{1}{R}(\vec{v}_{cm}\times\vec{\omega}).\int \vec{\eta} dm_2 - 3\frac{1}{R^3}\int \rho(\eta) d^{(3)}\eta\{(\vec{v}_{cm}\times\vec{\omega}).\vec{\eta}\}\{\vec{R}.\vec{\eta}\} + ...$$

$$= -\frac{Gm_1m_2}{R} - \frac{3Gm_1m_2\vec{v}_{cm}^{\ 2}}{R} - \frac{3Gm_1}{R^3}\int \rho(\eta) d^{(3)}\eta\{(\vec{v}_{cm}\times\vec{\omega}).\vec{\eta}\}\{\vec{R}.\vec{\eta}\} + ...$$

$$E_{12}^{non-recoil}(spin) = -\frac{3Gm_1}{R^3}\int \rho(\eta) d^{(3)}\eta\{(\vec{v}_{cm}\times\vec{\omega}).\vec{\eta}\}\{\vec{R}.\vec{\eta}\} \quad [neglecting\ higher\ order\ terms]$$

$$= -\frac{3Gm_1}{2R^3}(\vec{v}_{cm}\times\vec{\omega}).\vec{R}[\frac{2}{3}\int \eta^2 \rho(\eta) d^{(3)}\eta]$$

$$= -\frac{3Gm_1}{2R^3 m_2}(\vec{R}\times\vec{v}_{cm}).\vec{\omega}I_2$$ [I_2 = *moment of inertia of the 2nd body respect to any axis* passing *through its center*]

$$\boxed{E_{12}^{non-recoil}(spin) = -\frac{3Gm_1}{2R^3 m_2}(\vec{L}_2.\vec{S}_2)}$$ [$\vec{S}_2$ = *spin angular momentum*]

If we now take ***recoiling effect into account*** then it comes as,

$$E_{12}^{recoil}(spin) = 2Gm_1\frac{(\vec{R}\times\vec{v}_1).\vec{S}_2}{R^3} - \frac{3Gm_1}{2R^3 m_2}(\vec{R}\times m_2\vec{v}_2).\vec{S}_2$$

$$= -\frac{2G}{R^3}(\vec{R}\times\vec{P}).\vec{S}_2 - \frac{3Gm_1}{2R^3 m_2}(\vec{R}\times\vec{P}).\vec{S}_2 \quad [as\ \vec{v}_1 = \frac{\vec{p}_1}{m_1} = -\frac{\vec{P}}{m_1}\ \&\ \vec{v}_2 = \frac{\vec{p}_2}{m_2} = \frac{\vec{P}}{m_2}]$$

$$\boxed{\begin{aligned} E_{12}^{recoil}(spin) &= -G(2+\frac{3m_1}{2m_2})\frac{(\vec{R}\times\vec{P}).\vec{S}_2}{R^3} \\ &= -G(2+\frac{3m_1}{2m_2})\frac{\vec{L}_2.\vec{S}_2}{R^3} \end{aligned}}$$ [$\vec{L}_2$ = *orbital angular momentum of the 2nd body*]

<u>SPIN PRECESSION</u>:

Equation of motion of $\vec{S}_2$

$$\frac{dS_{2\ i}}{dt} = \{S_{2\ i}, H\}$$

[*for any function* $f(q,p,t)$

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial q}\frac{dq}{dt} + \frac{\partial f}{\partial p}\frac{dp}{dt}$$ H = *Hamiltonian of the system*

$$= \frac{\partial f}{\partial t} + \frac{\partial f}{\partial q}\frac{\partial H}{\partial p} - \frac{\partial f}{\partial p}\frac{\partial H}{\partial q}$$

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \{f, H\}$$

$$here, \frac{\partial S_{2\ i}}{\partial t} = 0]$$

So, due to ***gravitational spin-orbit coupling,*** the spin vector of an orbiting gyroscope will precess around the normal to the orbit which is given by

$$\frac{d\vec{S}_2}{dt} = \vec{\Omega}_{2\ precession} \times \vec{S}_2 \qquad [\vec{\Omega}_2 = angular\ velocity\ of\ 2nd]$$

where,

$$\vec{\Omega}_{2\ precession}^{non-recoil} = \frac{3Gm_1}{2R^3 m_2}\vec{L}_2 \qquad [c=1]$$

$$\vec{\Omega}_{2\ precession}^{recoil} = G(2+\frac{3m_1}{2m_2})\frac{1}{R^3}\vec{L}_2 \qquad [c=1]$$

$$\vec{\Omega}_{2\ precession}^{non-recoil} = \frac{3Gm_1}{2R^3 m_2}\vec{L}_2\frac{1}{c^2} \qquad [c=3\times10^8\ m\,s^{-1}]$$

$$\vec{\Omega}_{2\ precession}^{recoil} = G(2+\frac{3m_1}{2m_2})\frac{1}{R^3}\vec{L}_2\frac{1}{c^2} \qquad [c=3\times10^8\ m\,s^{-1}]$$

above two results are obtained by assuming $\mathrm{m}_1 >> m_2$

& only 2nd body is spinning respect to its own axis.

$$\Uparrow$$

$$\boxed{RELATIVISTIC\ SPIN-ORBIT\ COUPLING}$$

For sun-earth system

Assumption:

1) only earth (m_2) is spinning
2) Earth is perfectly spherical in shape

Then also the spin axis of the earth does precess around the normal to the ecliptic with above frequency whereas according to Newtonian Gravity this shouldn't precess at all under above conditions. Frequency of this relativistic precession ***due to spin-orbit coupling*** is

$$\vec{\Omega}_{precession}^{recoil}(EARTH) = 6.67\times10^{-11}[2+\frac{3\times(2\times10^{30})}{2\times(5.98\times10^{24})}]\frac{1}{(149.6\times10^9)^2}$$

$$[(5.98\times10^{24})(29.8\times10^3)\frac{1}{(3\times10^8)^2}]$$

$$[v_E \approx 29.8\times10^3\ m\,s^{-1}\ \text{and assuming}\ \theta\approx 90^\circ]$$

$$= 2.96\times10^{-15}\ rad\ s^{-1}$$

Therefore, to complete one complete revolution around the axis perpendicular to ecliptic spin axis of the earth will take

$$\frac{2\pi}{2.96\times10^{-15}}\quad \sec$$
$$=\frac{2\pi}{2.96\times10^{-15}\times(365\times24\times3600)}\quad years$$
$$=6.73\times10^{7}\quad years$$

$$\boxed{Time\ period=6.73\times10^{7}\ years}\Leftarrow\boxed{RELATIVISTIC\ SPIN-ORBIT\ COUPLING}$$

NOTE: Here I haven't taken the effect due to moon into account because to apply above formula the condition $m_1 >> m_2$ should be satisfied, whereas in case of moon the mass of the moon is less than that of the earth.

Binary pulsar system

Above equations are also applicable for binary star system only when one of the stars is very massive compared to the other. But if in a binary system two stars have almost same mass (e.g. PSR J0737-3039A/B binary pulsar system) then we need a small modification of the above formula that is

$$\boxed{\begin{array}{l}\vec{\Omega}_{2\ precession}^{recoil}(Double\ pulsar)=G\left(2+\frac{3m_1}{2m_2}\right)\frac{m_1m_2}{m_1+m_2}\frac{1}{R^3}\vec{R}\times\vec{v}_2\frac{1}{c^2}\qquad[c=3\times10^{8}\ m\,s^{-1}]\\ when\ only\ 2nd\ body\ is\ spinning\ respect\ to\ its\ own\ axis.\end{array}}$$

$$\Uparrow$$

$$\boxed{RELATIVISTIC\ SPIN-ORBIT\ COUPLING}$$

$$\vec{\Omega}_{2\ precession}^{recoil}(PSR1913+16) \approx G(2+\frac{3}{2})\frac{m}{2}\frac{1}{R^2c^2}v_2 \qquad [\theta \approx \frac{\pi}{2}\ \&\ m_1 \approx m_2 = m]$$

$$= 6.67\times10^{-11}\times\frac{7}{2}\times\frac{1.4\times2\times10^{30}}{2}\times\frac{1}{(4.8\times6.955\times10^8)^2(3\times10^8)^2}\times75\times10^3 \quad rad\ s^{-1}$$

$$[at\ APASTRON\ position\ \ R \approx 4.8R_\odot\ \ v_2 \approx 75\,km\,s^{-1}]$$

$$= 2.44\times10^{-11}\ \ rad\ s^{-1}$$

$$Therefore\ time\ period\ of\ precession = \frac{2\pi}{2.44\times10^{-11}\times(3600\times24\times365)}\ years$$

$$\approx 8153\ \ years$$

⇑

AT APASTRON POSITION OF PSR1913+16

$$\vec{\Omega}_{2\ precession}^{recoil}(PSR1913+16) \approx G(2+\frac{3}{2})\frac{m}{2}\frac{1}{R^2c^2}v_2 \qquad [\theta \approx \frac{\pi}{2}\ \&\ m_1 \approx m_2 = m]$$

$$= 6.67\times10^{-11}\times\frac{7}{2}\times\frac{1.4\times2\times10^{30}}{2}\times\frac{1}{(1.1\times6.955\times10^8)^2(3\times10^8)^2}\times300\times10^3 \quad rad\ s^{-1}$$

$$[at\ PERIASTRON\ position\ \ R \approx 1.1R_\odot\ \ v_2 \approx 300\,km\,s^{-1}]$$

$$= 1.86\times10^{-9}\ \ rad\ s^{-1}$$

$$Therefore\ time\ period\ of\ precession = \frac{2\pi}{1.86\times10^{-9}\times(3600\times24\times365)}\ years$$

$$\approx 107\ \ years$$

⇑

AT PERIASTRON POSITION OF PSR1913+16

Therefore, average time period of the precession of the spin axis of the pulsar is ~4130 years.

- SPIN-SPIN INTERACTION

There is another type of effect which is spin-spin interaction. When both of the objects spin with respect to own axis of symmetry then this phenomena comes into account. This also leads to the spin precession.

$$E_{12}(spin-spin\ \text{interaction}) = \frac{G}{R^5}[3(\vec{S}_1.\vec{R})(\vec{S}_2.\vec{R})-(\vec{S}_1.\vec{S}_2)R^2]$$

Therefore after taking both effects into account the angular velocity of the spin precession becomes

$$\boxed{\vec{\Omega}_2 = [G(2+\frac{3m_1}{2m_2})\frac{m_1 m_2}{m_1+m_2}\vec{R}\times\vec{v}_2\frac{1}{c^2}]+[\frac{G}{R^5}\{3(\vec{S}_1,\vec{R})\vec{R}-R^2\vec{S}_1]}$$

and

$$\boxed{\vec{\Omega}_1 = [G(2+\frac{3m_2}{2m_1})\frac{m_1 m_2}{m_1+m_2}\vec{R}\times\vec{v}_1\frac{1}{c^2}]+[\frac{G}{R^5}\{3(\vec{S}_2,\vec{R})\vec{R}-R^2\vec{S}_2]}$$

D. **CONCLUSION**

Therefore, in case of sun-earth system

$$T^{EARTH}_{NEWTONIAN} \approx 25675\ years = 2.57\times10^4\ years$$
$$T^{EARTH}_{RELATIVISTIC} \approx 6.73\times10^7\ years$$
Therefore,
$$\frac{T^{EARTH}_{RELATIVISTIC}}{T^{EARTH}_{NEWTONIAN}} = 2.62\times10^3$$

$\Leftarrow$ EFFECT DUE TO SUN

the Newtonian precession is dominant over general relativistic effect.

Another thing is that I can apply the general formula

$$\vec{\Omega}_{2\ precession}^{recoil} = G(2+\frac{3m_1}{2m_2})\frac{m_1 m_2}{m_1+m_2}\frac{1}{R^3}\vec{R}\times\vec{v}_2\frac{1}{c^2} \qquad [c = 3\times10^8\ m\,s^{-1}]$$

when only 2nd body is spinning respect to its own axis.

$\Uparrow$

RELATIVISTIC SPIN – ORBIT COUPLING

to calculate the effect due to the moon on the spin axis of the earth.

$$\vec{\Omega}^{recoil}_{EARTH\ \ precession} \approx 6.67\times10^{-11}[2+\frac{3\times7.38\times10^{22}}{2\times5.98\times10^{24}}][\frac{(7.38\times10^{22})(5.98\times10^{24})}{(7.38\times10^{22})+(5.98\times10^{24})}]$$

$$\frac{1}{(384.4\times10^{6})^{2}}\frac{29.8\times10^{3}}{(3\times10^{8})^{2}} \qquad [\text{assuming}\ \theta\approx 90^{\circ}]$$

$$=2.2\times10^{-17}\ \ rad\ s^{-1}$$

Therefore, corresponding time period

$$\boxed{T^{EARTH}_{RELATIVISTIC} \approx 9.06\times10^{9}\ \ years} \Leftarrow \boxed{EFFECT\ DUE\ TO\ MOON}$$

Time period is 100 times larger than that of the sun which is expected due to heavy mass of the sun ($\frac{m_{sun}}{m_{moon}} \sim 10^{8}$).

For binary pulsar system

$$\boxed{T^{PULSAR}_{RELATIVISTIC} \approx 4130\ years}$$

though according to Newtonian gravity the spin axis shouldn't precess at all if we assume them as perfectly spherical in shape then also this does precess around the normal to the orbital plane. If in a binary pulsar system each of them is a pulsar and both have comparable mass then spin axis of ***both*** pulsar precess around the common normal to the orbital plane.

Another point is to be noted that ***relativistic*** rate of precession of spin axis of pulsar is far greater than that of earth which is expected.

Though to figure out the precession rate I have taken some approximations then also the results I obtained are very much closed to the observational outcomes. I have assumed the earth-sun as one isolated system i.e. there is only existence of mutual gravitational force between earth & sun, but actually there is gravitational force acting on sun as well as earth by other astronomical objects present in the rest of the universe since gravitational force is infinitely extent. As sun is too massive compared to other astronomical objects present in the solar system, so the center of mass of whole solar system is lying inside the sun very closed to the center of the sun and the orbital velocity of the sun is almost negligible compared to that of other objects in solar system. So the *non-recoiling* approximation is a good one for this. But in case of double pulsar system this is not a good one because here both have comparable mass, so the center of mass of the double pulsar system actually lies almost at equal distance from two.

Therefore both have almost comparable velocity. Another point is that throughout the calculation I have neglected the *retardation* effect. I have used same time in r(t) and r'(t) which denotes they interact instantaneously but according to theory of relativity no signal can propagate faster than the speed of light in vacuum. So actually I neglect the time taken by gravitons to reach at another body from one. But near the binary pulsar system this effect actually plays a major role due to heavy mass of pulsar.

The source theory approach actually can describe many phenomena successfully but based on which this theory actually develops is not experimentally found, that is GRAVITON. Here though the geometrical complication like Einstein's theory of general relativity is absent but it doesn't give us much information regarding '*time'* which plays a vital role in GTR. Therefore, though it is comparatively easier to handle this approach than GTR but this is actually lack of notion of 'time'.

Therefore, through this project the important message which I want to deliver is that the notion of the torque & so the precession in Newtonian Mechanics is not sufficient to describe some natural phenomena, instead torque can be present due to some different reason, that may be spin-orbit coupling or may be spin-spin coupling and so the precession can be occurred though Newtonian gravity says this shouldn't occur at all!!

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